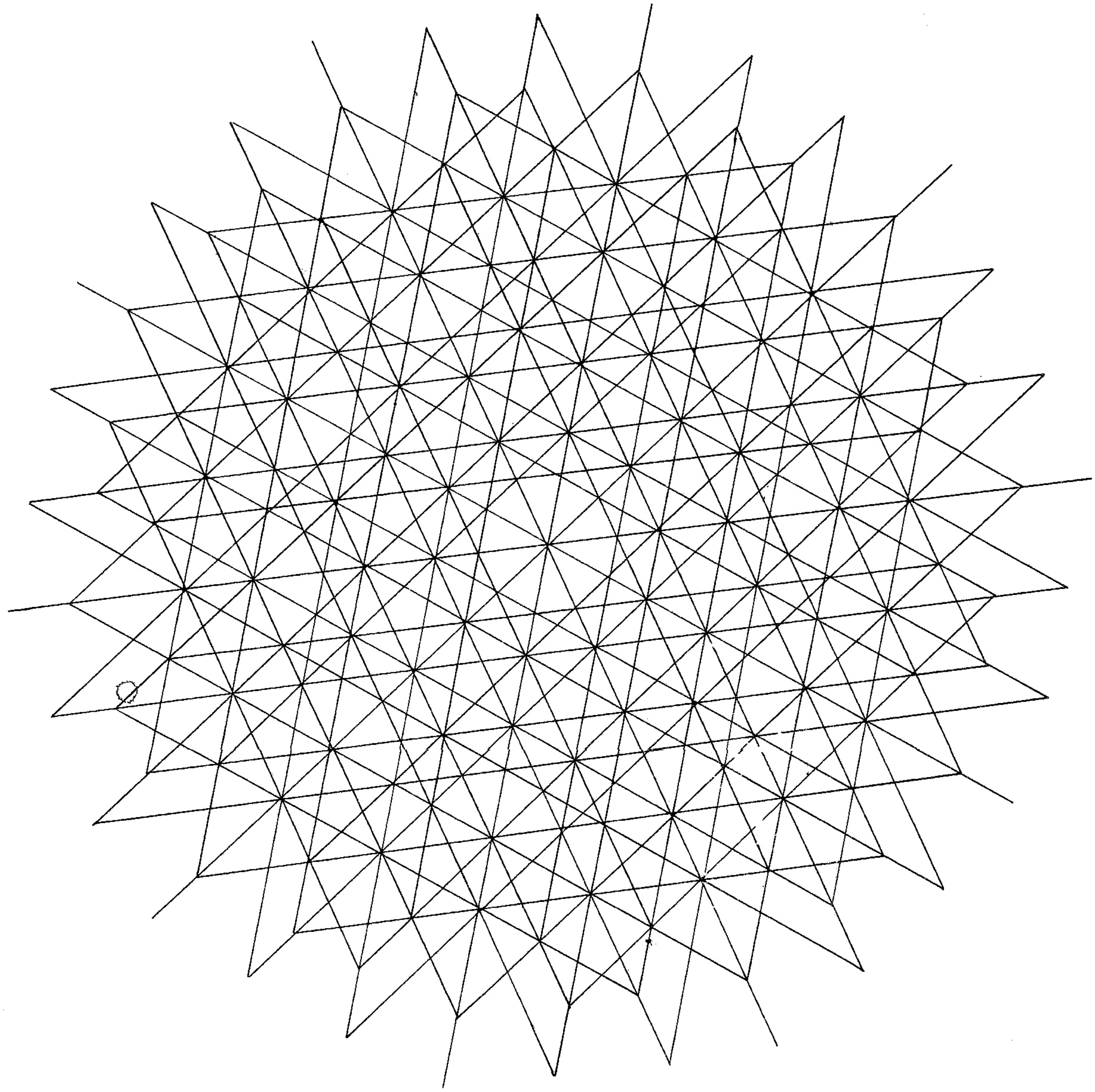

zome primer

by Steve Baer

elements of zonohedra geometry

two and three dimensional growths of stars with five fold symmetry

THE BOOK THAT RUINED MY LIFE ....

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Zome Primer

Zome Primer
Elements of Zonohedra Geometry

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Zomeworks Corporation | Albuquerque, New Mexico

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To

My Wife Holly

Preface

This booklet has first an elementary explanation of the geometry of Zonohedra, then a more difficult account of the growths of the thirty-one zone star. This system, based on the 31 lines that pass through the center of an icosahedron and either a vertex, edge midpoint or face midpoint is new and unusual.

I have applied for a patent on this structural system. The patent is assigned to Zomeworks Corporation. The predecessors of this system are the octet truss and the MERO space grid system. The relative potentials of these systems are discussed briefly by a comparison of their geometric possibilities.

The forms possible using this system are limitless; there is no attempt here to explore these possibilities—the examples shown are small probings. The booklet describes the mathematics of the process that creates these limitless forms.

The framework for the Robert Ford residence was designed by Jim Welty and Robert Ford.

The shallow rectangular based trusses were designed by Berry Hickman of Zomeworks who also introduced the excellent plastic ball joint used throughout in the models photographed.

The design and manufacture of the 6-zone aluminum joint was done by Otto Jung of Design Industries in Albuquerque.

Ken Leonard did the layout and both he and my wife, Holly, gave much editorial assistance.

S.C.B.
Albuquerque
August 1970

Contents

1 Zomes, Domes and Clusters

1.1 What Are Zomes?

A Zome is a man-made structure derived from zonohedra. The zones of the zonohedron may be stretched or shrunk or removed to produce, if desired, an asymmetric dome shaped structure.

Zomes may be single or clustered.

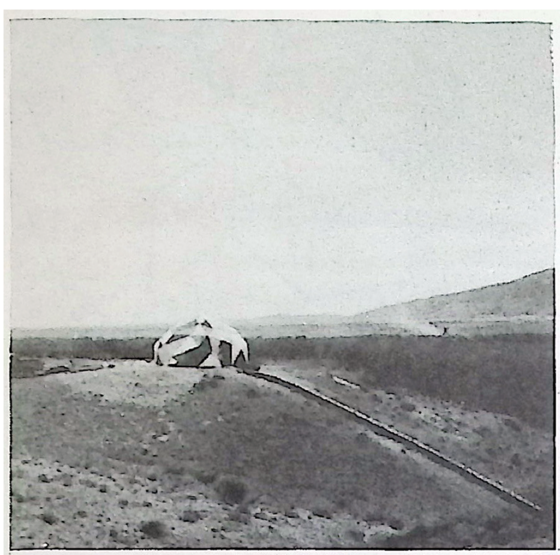


Figure 1: Zome

Figure 2: Zome, with person and child

Zomes can cluster together like soap bubbles. Their zones can be stretched, shrunk, or omitted completely to make the various zomes' different shapes and sizes. The zomes can also pack several layers deep.

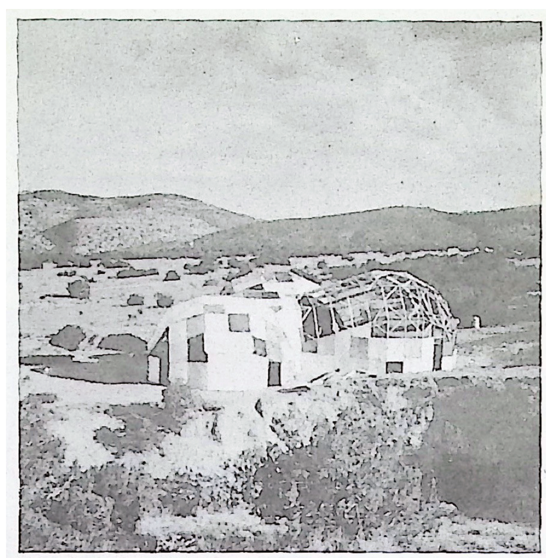


Figure 3: Construction of a Zome structure

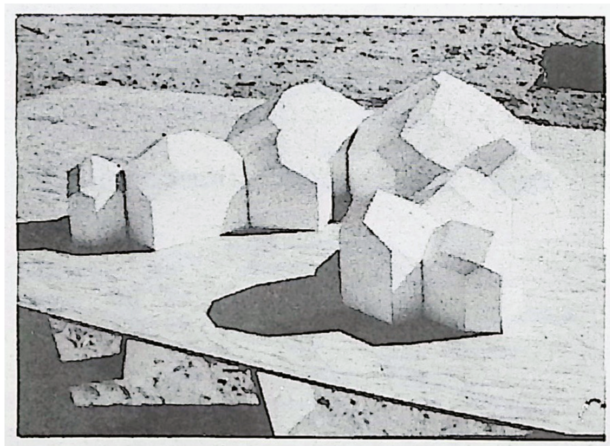


Figure 4: Scale model of a Zome structure

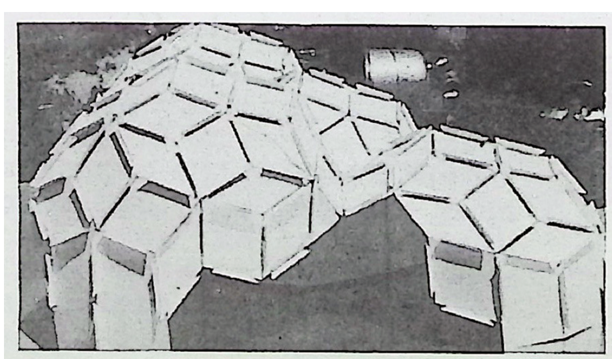


Figure 5: Construction of a Zome structure

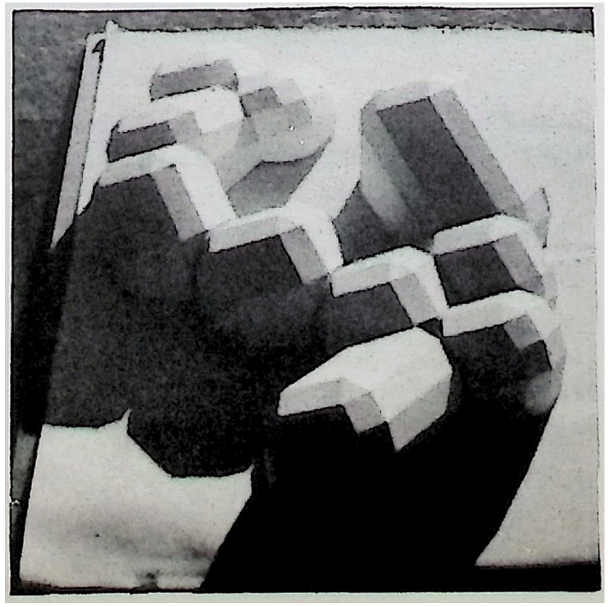


Figure 6: Scale model of a Zome structure

1.2 What is the Difference Between a Zome and a Geodesic Dome?

A geodesic dome is a structure which closely follows the shape of the sphere and whose edge lengths closely follow the path of great circles on the sphere. (These are the sphere's geodesics.) The geodesic dome, because of its shape, and the arrangement of its structural members is extremely strong, but its uses are limited because of the inflexibility of its shape. It is always part of a sphere—a low bubble or a high bubble—its floor is always a circle—any variation would destroy the structural properties of the geodesic dome. The geodesic dome, if it is large and composed of many edges and joints, has many different edge lengths. It is complicated in structure and simple in shape. Zomes are simple in structure and complicated in shape.

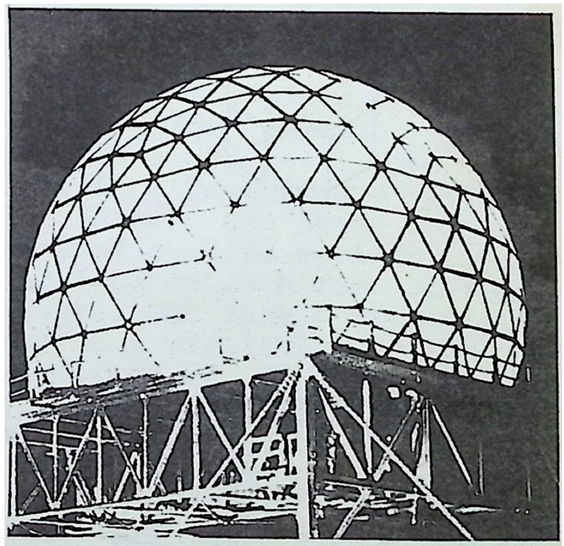


Figure 7: Geodesic Dome—DEW line Radome, circa 1956

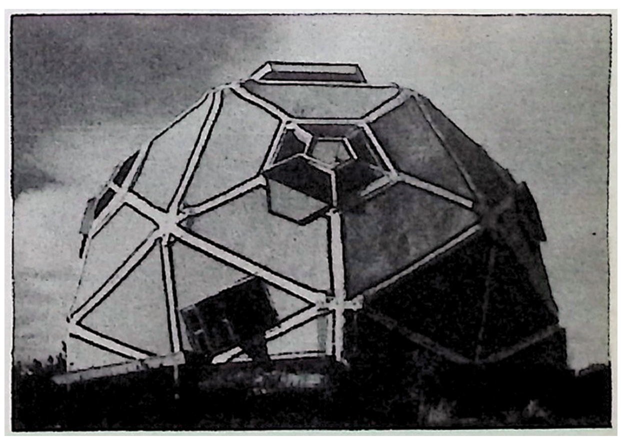


Figure 8: Geodesic Dome

2 Zonohedra

2.1 What are Zonohedra?

A zonohedron is a convex solid, all of whose faces are polygons with edges in equal and parallel pairs.

These are possible faces for zonohedra:

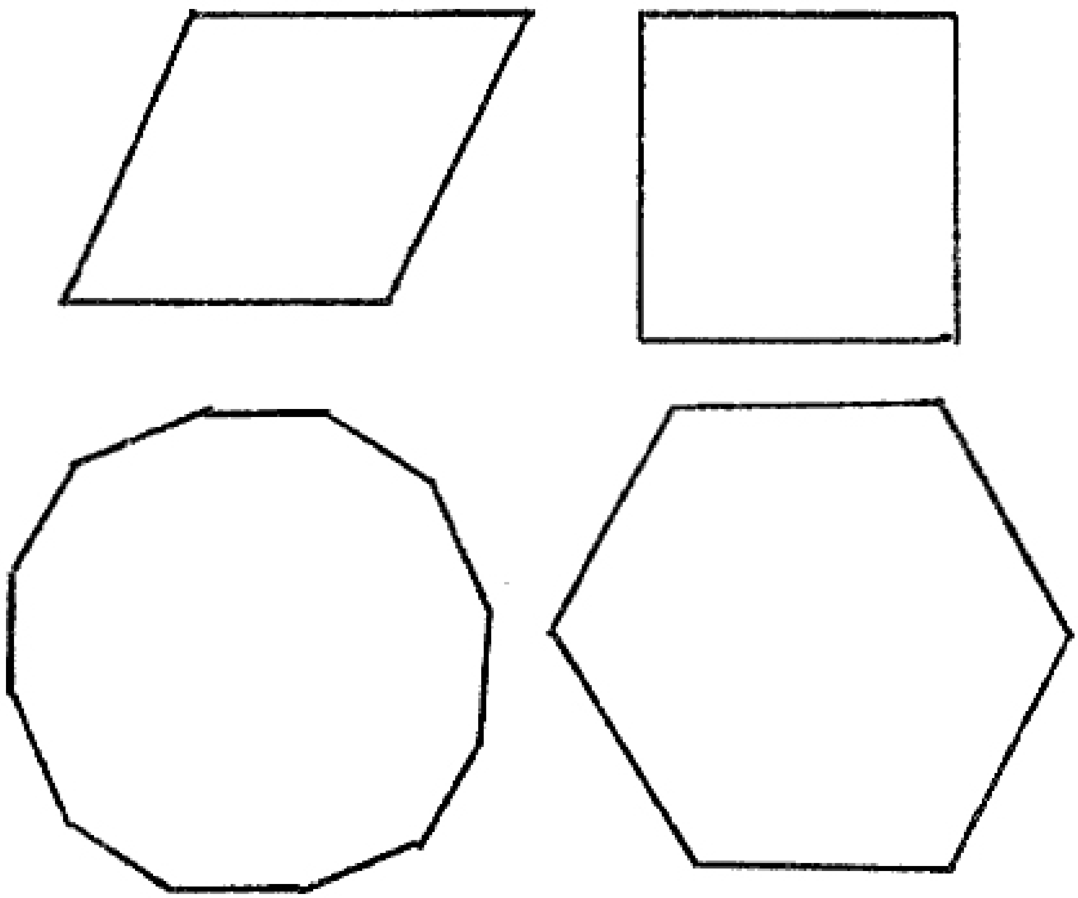


Figure 9: Possible faces

These are zonohedra:

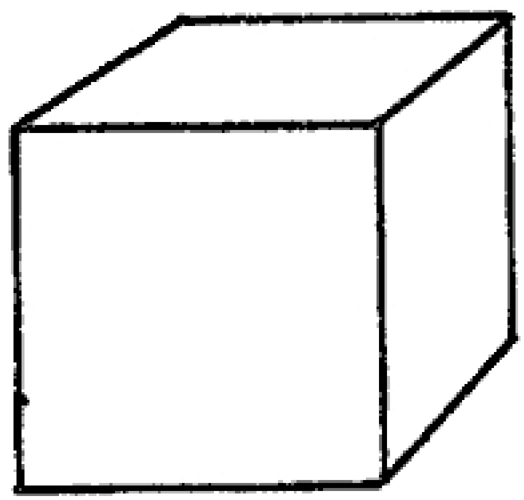


Figure 10: Cube

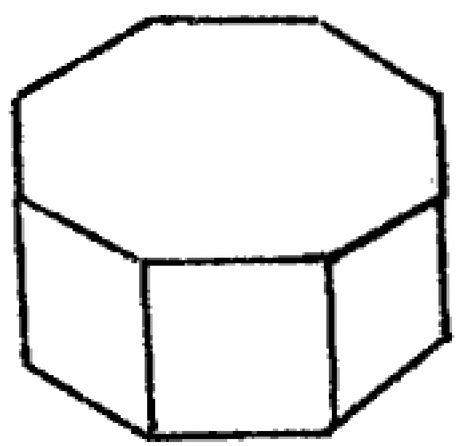


Figure 11: Octagonal prism

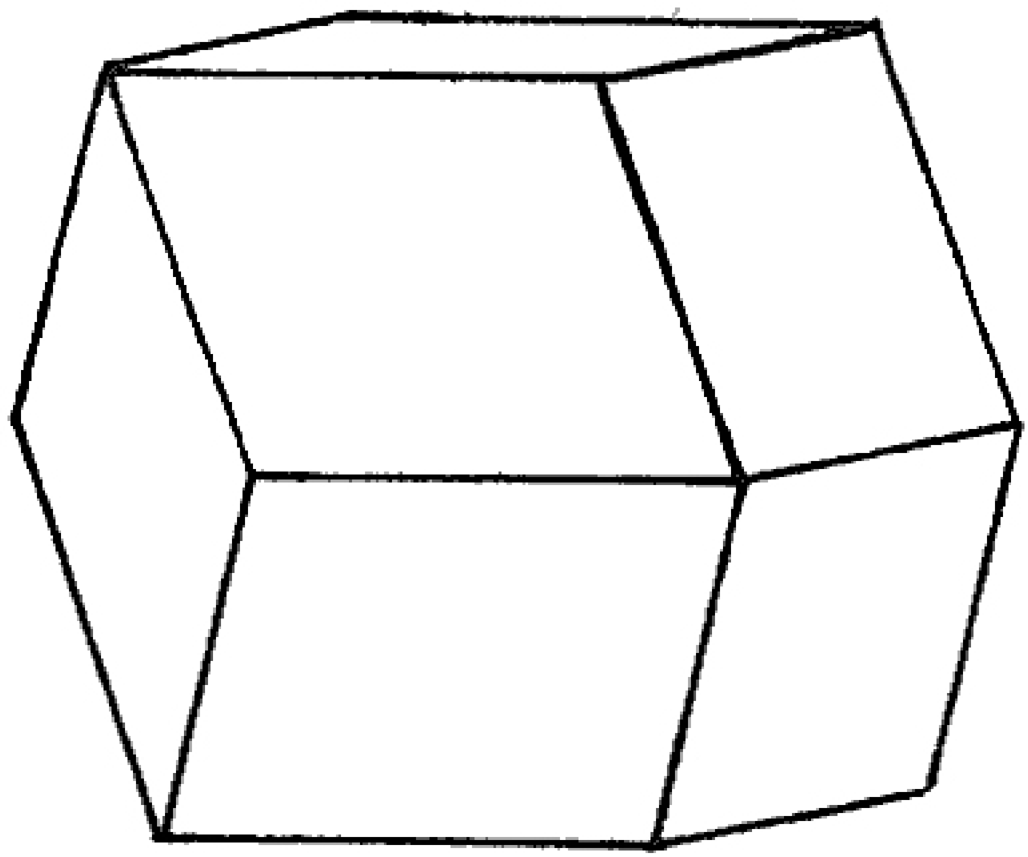


Figure 12: Rhombic Dodecahedron

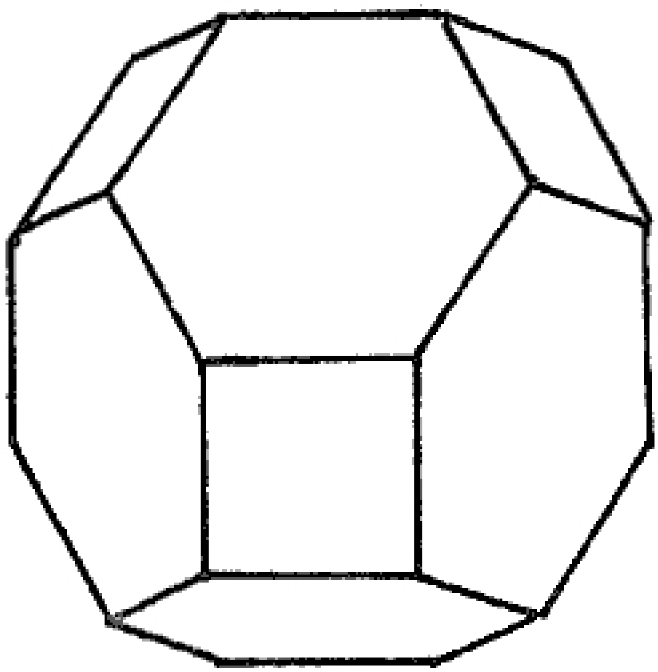


Figure 13: Truncated Octahedron

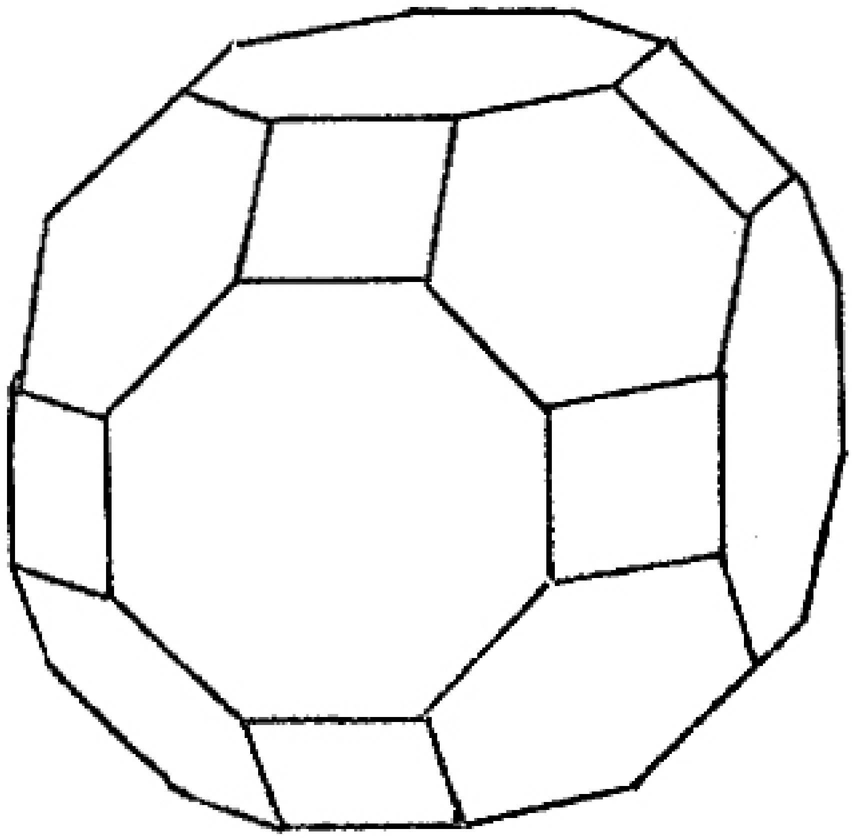


Figure 14: Truncated Cuboctahedron

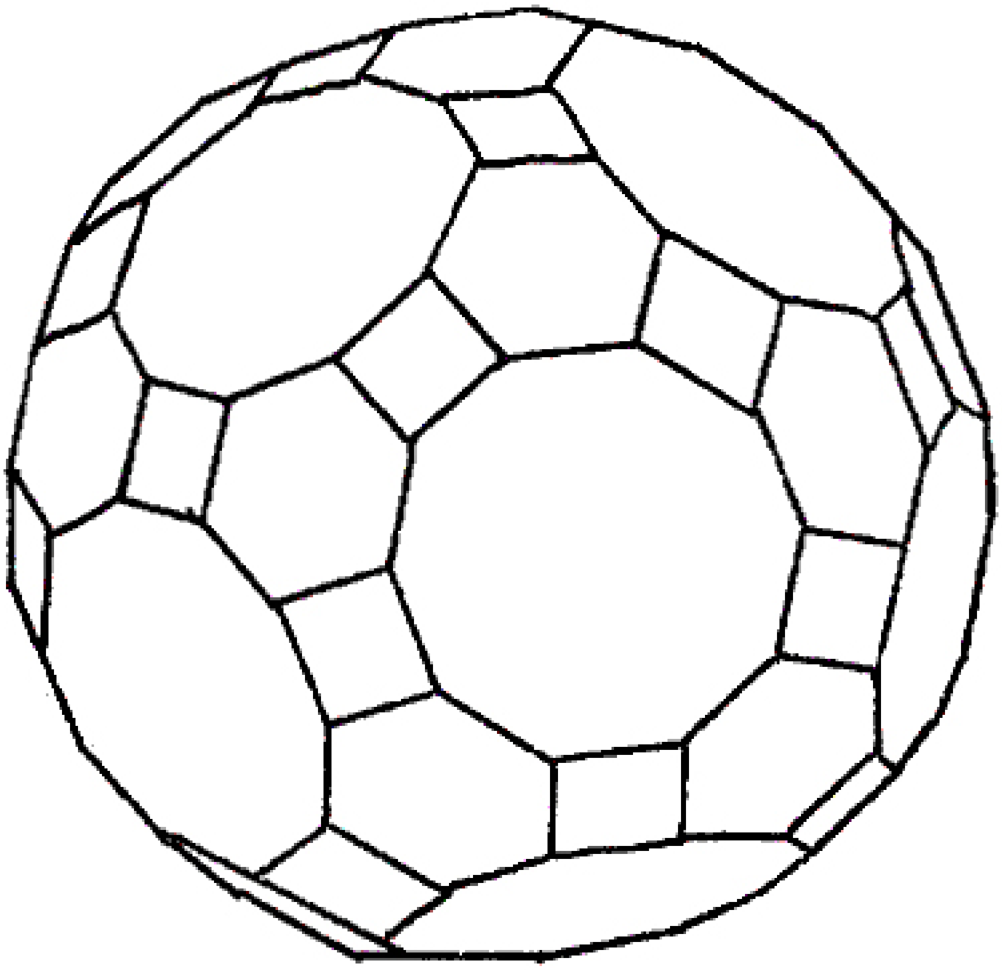


Figure 15: Great rhombicosidodecahedron or Truncated icosidodecahedron

A zone of edges is a band of parallel edges which circles the solid. Every edge belongs to a zone.

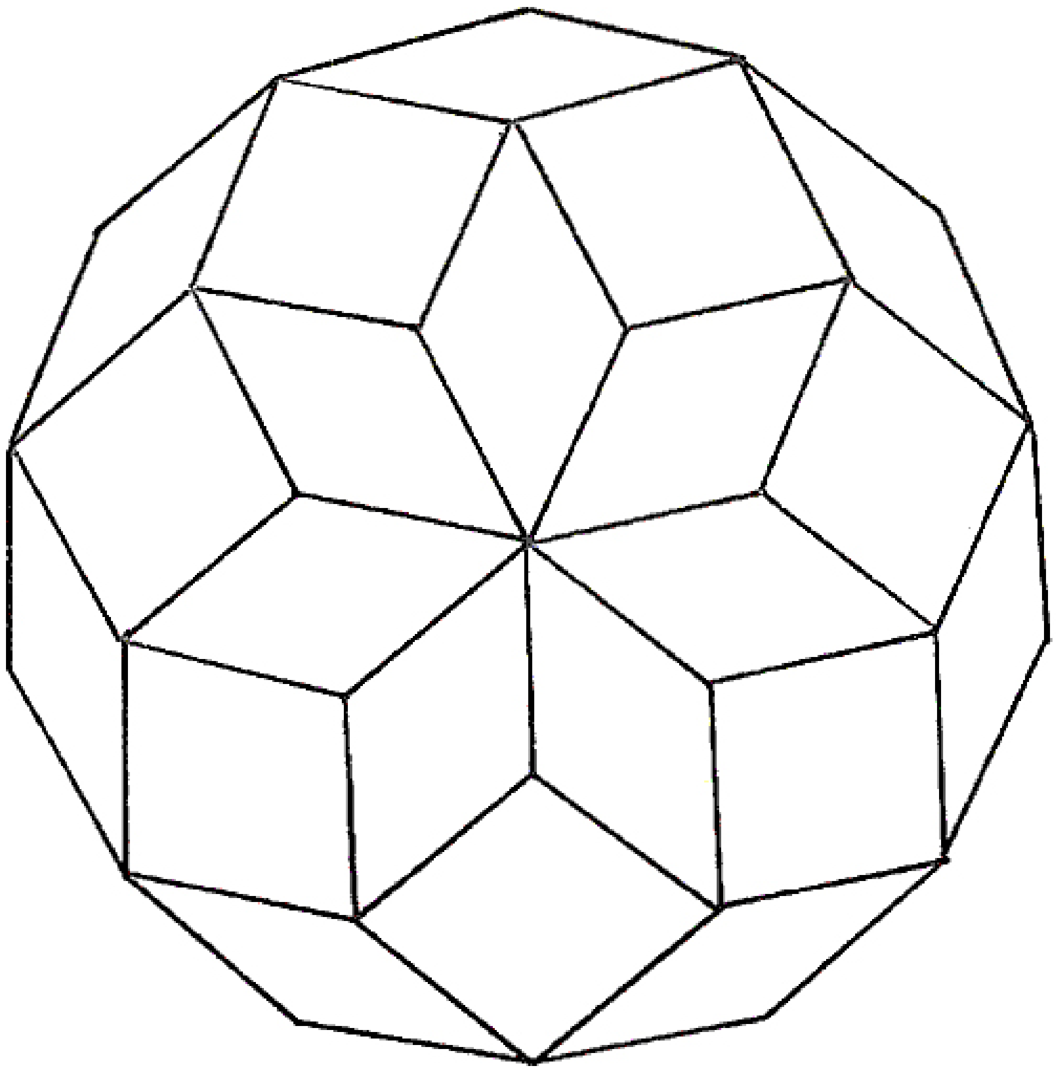


Figure 16: Seven-Zone Polar Zonohedron

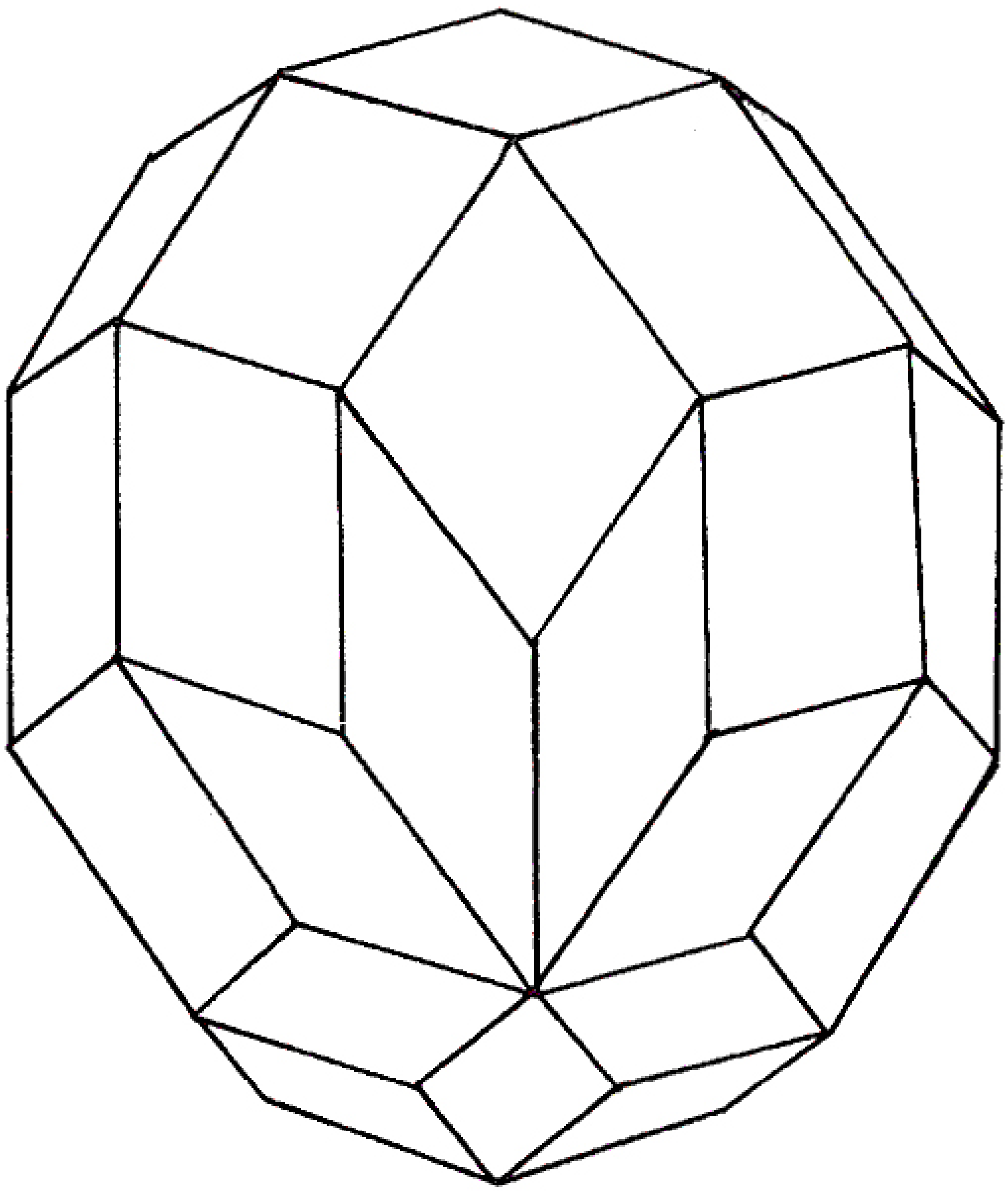


Figure 17: Polar Zonahedron from CHH Franklin’s drawing

2.2 Face Planes of Zonohedra

A plane is defined by two lines. A six-zone figure has six different lines; 1, 2, 3, 4, 5, 6. How many pairs can we form with six objects?

1,2	2,3	3,4	4,5	5,6	
1,3	2,4	3,5	4,6		
1,4	2,5	3,6			= 15
1,5	2,6				
1,6					

Table 2.1: Pairs of Six Objects

Algebraic expression:

$$X = \frac{(n)(n-1)}{2}$$

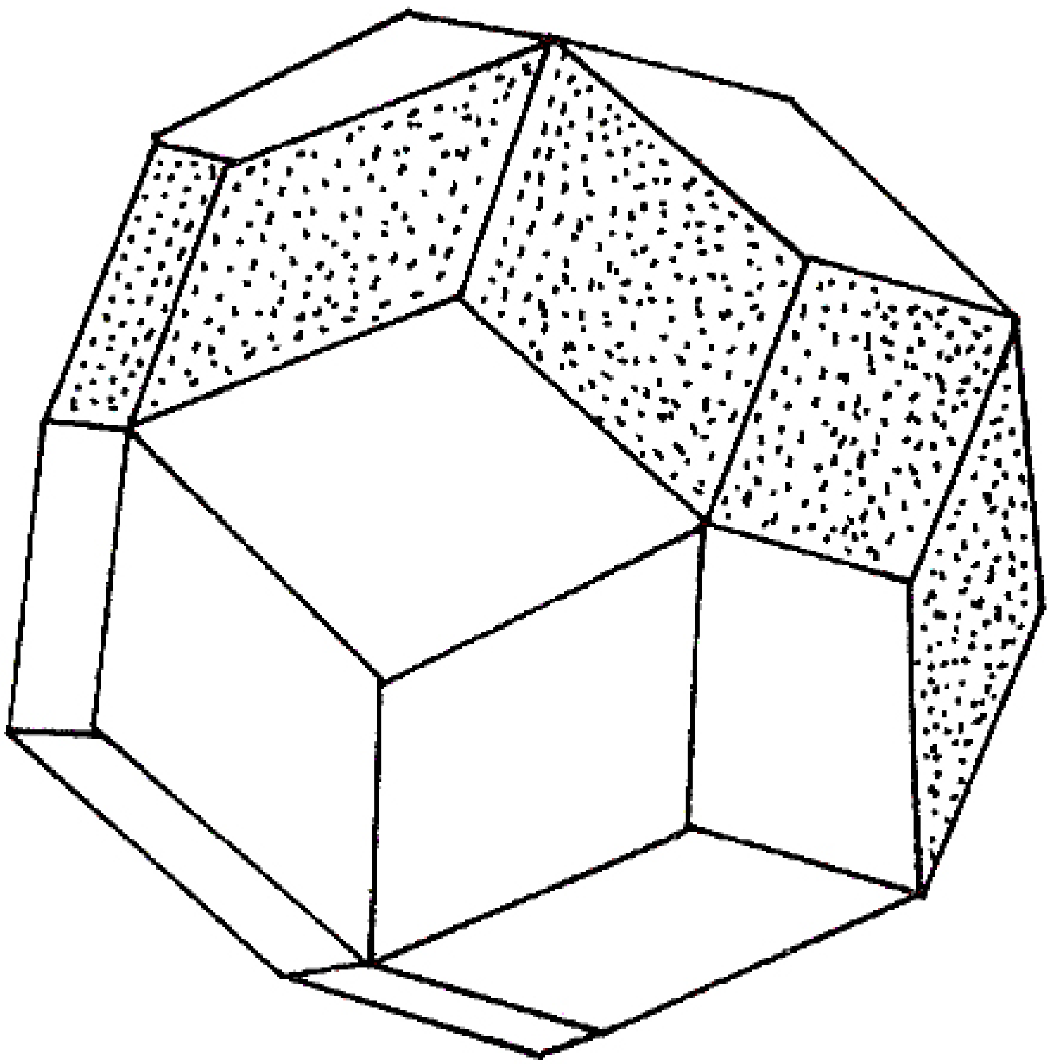


Figure 18: Zone “1” shaded

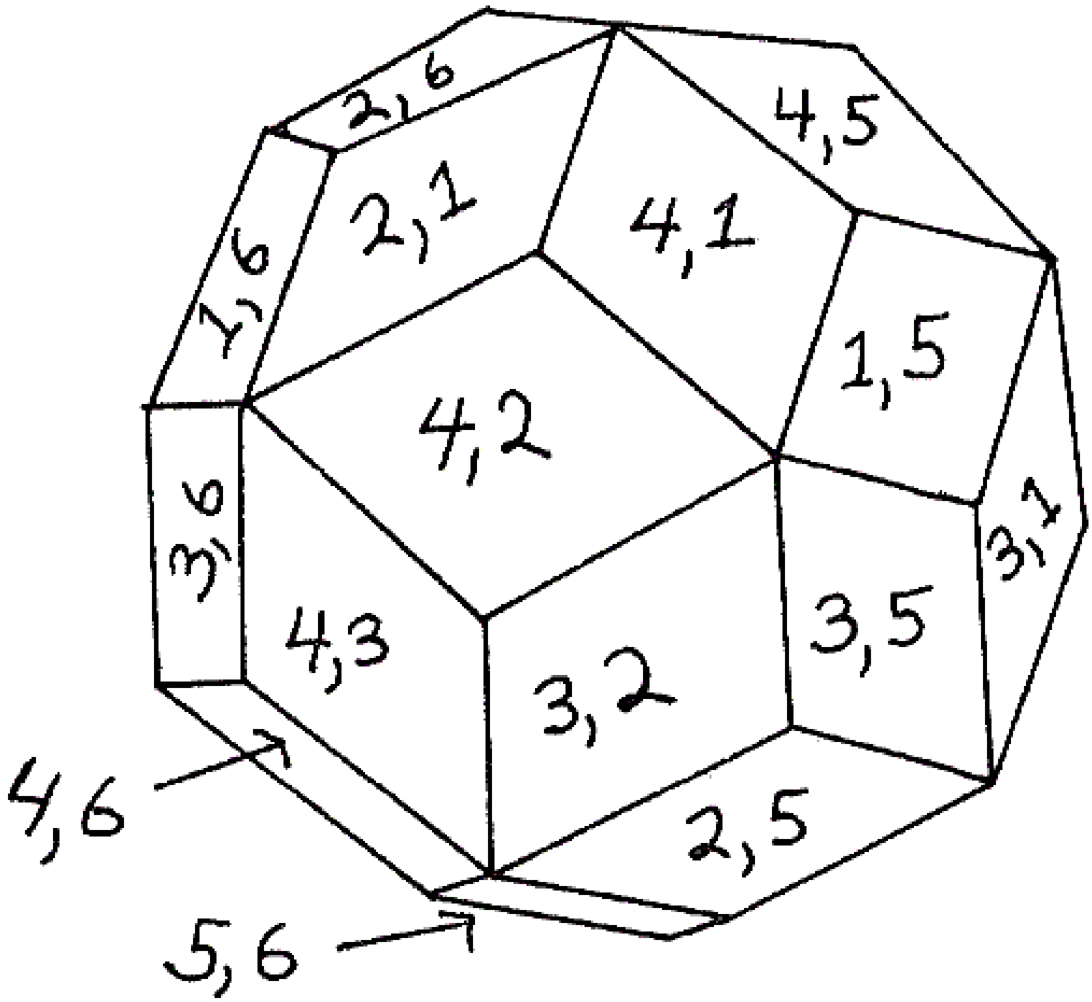


Figure 19: Rhombic Triconntahedron, labeled zones

Rhombic Triacontahedron with face planes labeled with numbers of the zones which form the plane.

There are then 15 more faces on the other side which add up to the 30 faces of the Triacontahedron.

The ten-zone system can form $\dfrac{10 \times 9}{2 \times 1} = 45$ different planes.

This figure is the enneacontahedron with its faces marked:

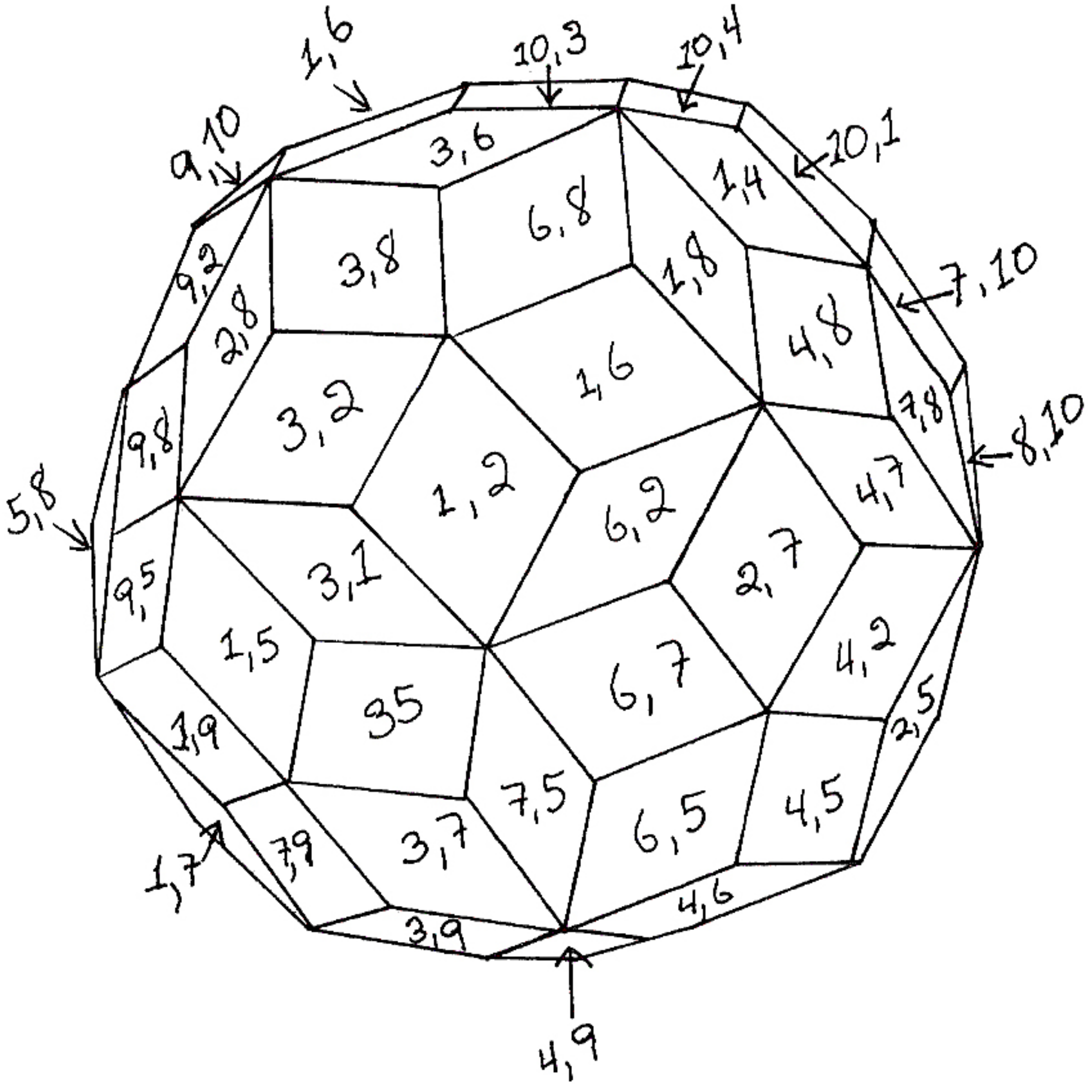


Figure 20: Enneacontahedron, labeled

The equation is for the number of planes, and if we wish to find the total number of faces for the polyhedron, including the back side, multiply the number of different planes by 2, and this equals $(n)(n-1)$.

The formula $\frac{(n)(n-1)}{2}$ is true for the number of planes that can be formed with the different zones provided that no more than two lines lie in one plane. These collections of lines associated with zonohedra are called the *stars* of the *zonohedra*. A star is called non-singular if no three of the lines are coplanar.

If three lines of the star lie in one plane, then the zonohedron associated with the star has a pair of hexagons. If more than three lines lie in one plane, then there are facets to the zonohedron with corresponding more edges—octagon, decagon, etc.

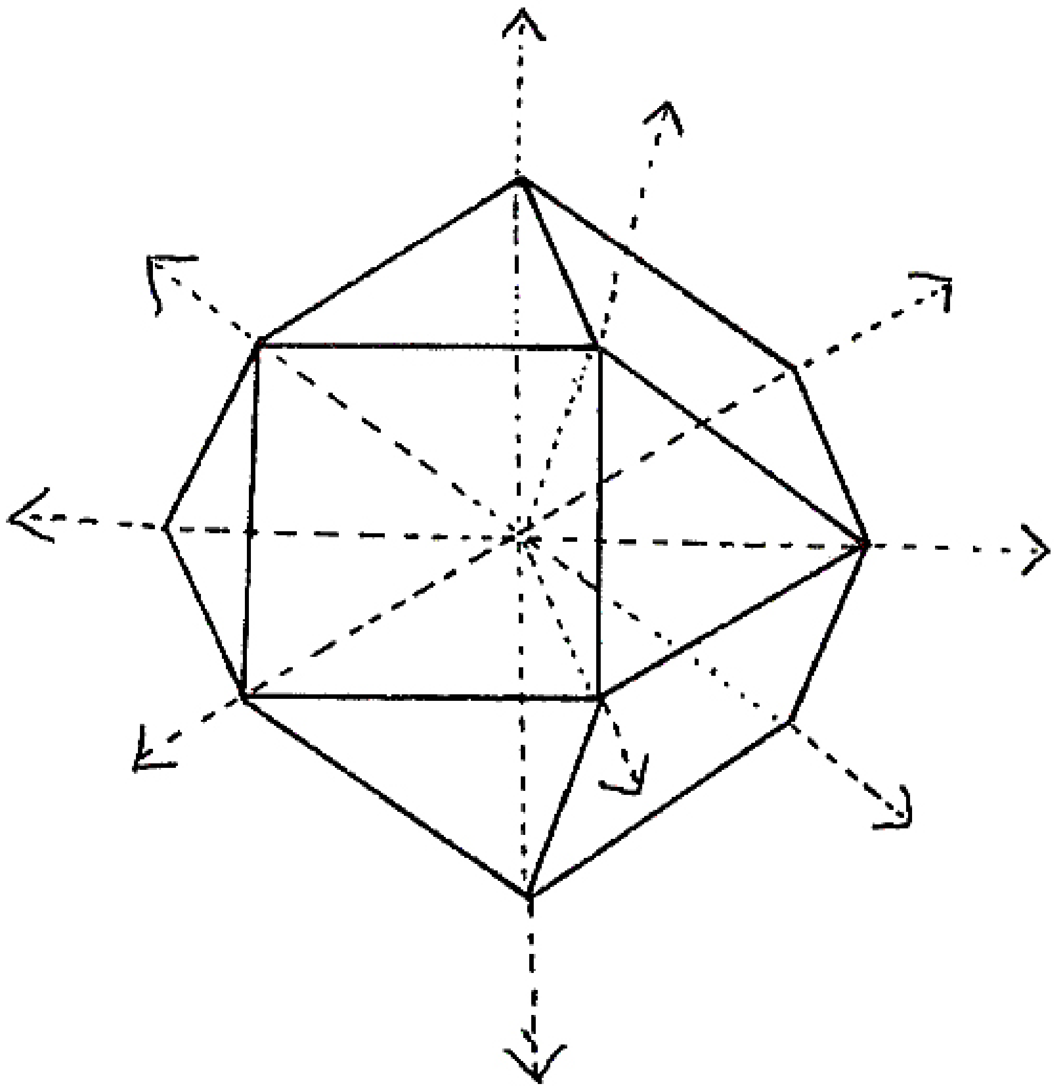


Figure 21: Singular Star, through vertices of cuboctahedron.

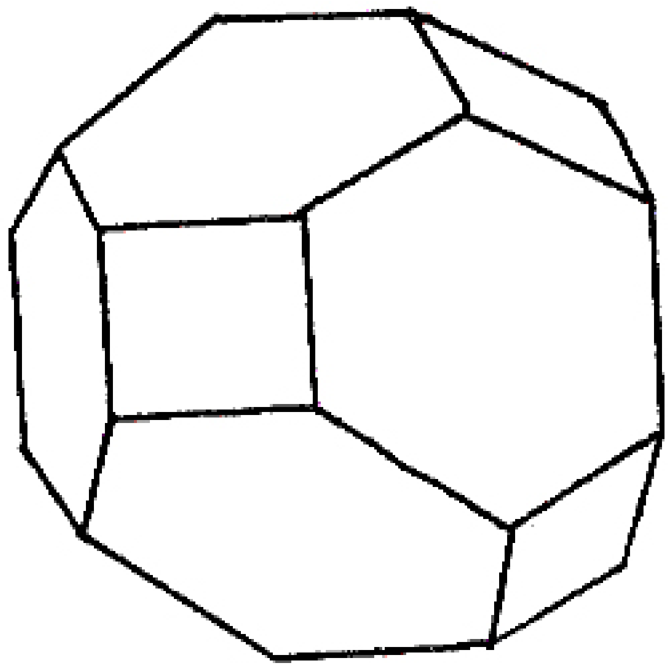


Figure 22: Associated zonohedron, truncated octahedron

2.3 Section Stars and Face Planes

The zonohedron that has its edges parallel with the lines of the 31-zone star is a huge figure with faces—two faces for each of the 121 sections—one face on each side of the figure. The face corresponding to a particular section is formed by the lines of the star following each other head to toe around in a complete polygon. Consequently, the 242 sided zonohedron associated with the 31 zone star has:

12	regular decagons	T sections
30	irregular dodecagons	R sections
60	irregular hexagons	S sections
20	regular hexagons	V sections
60	rectangles	X sections
60	rectangles	Y sections
242		

Table 2.2: Faces of a 31 Zone Star

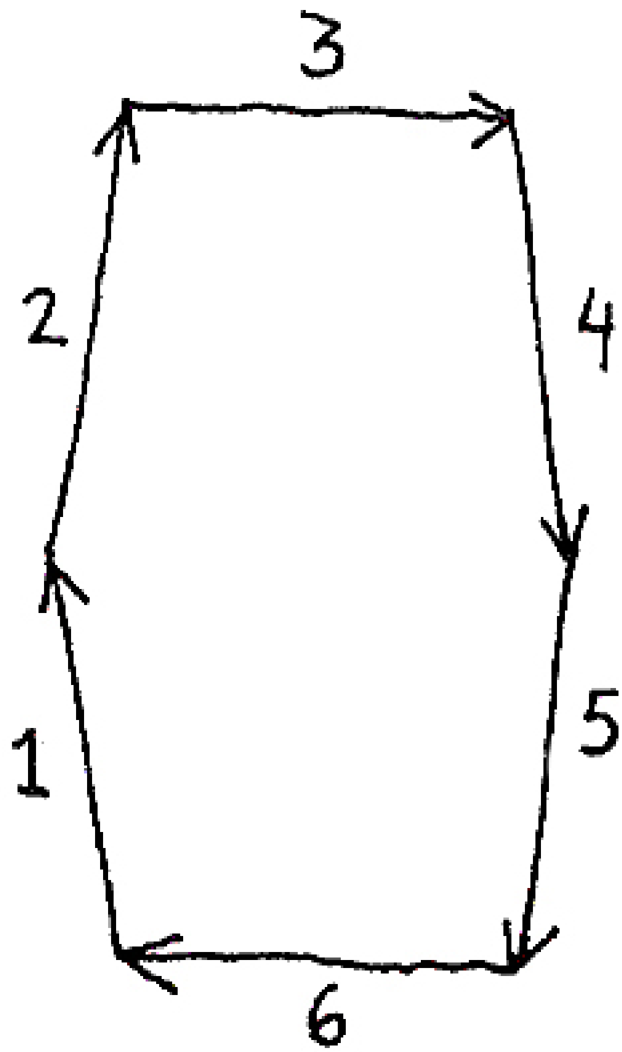


Figure 23: Associated face plane

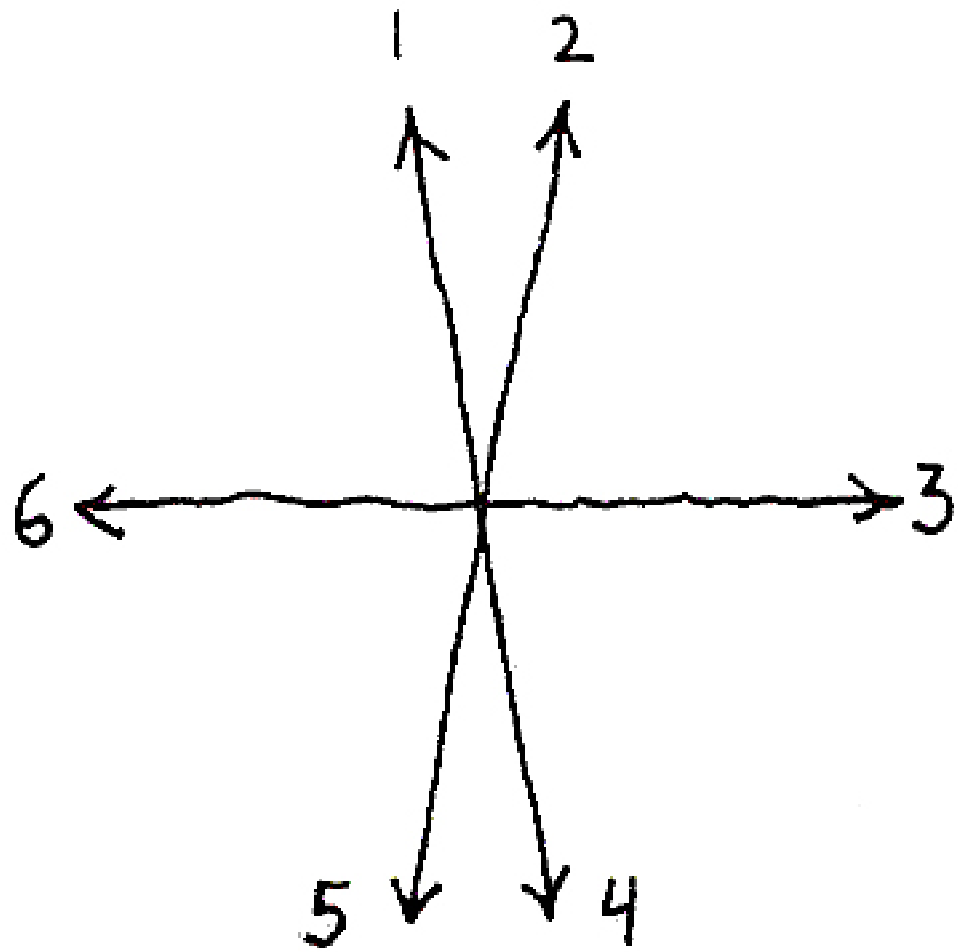


Figure 24: Section star

2.4 Division of Zonohedra into Parallepiped Cells

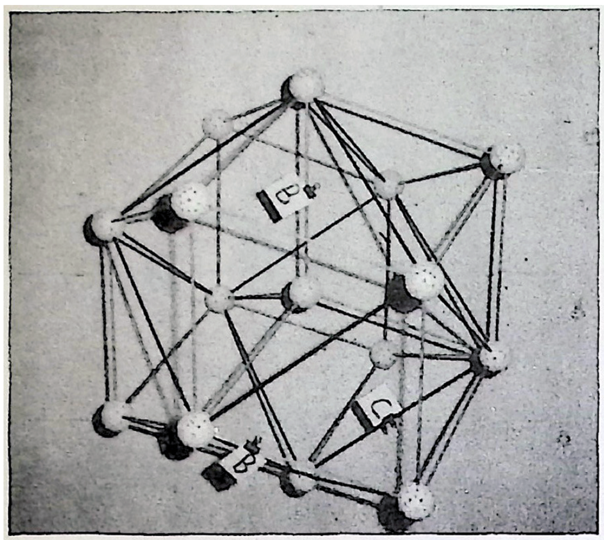


Figure 25: Rhombic Dodecahedron

Every zonohedron can be divided into component parallelepiped cells. Every set of three different lines form one cell. In the case of the four-zone rhombic dodecahedron 25, there are then:

$$C_3^4 = \frac{4 \cdot 3 \cdot 2}{3 \cdot 2 \cdot 1} = 4 \text{ cells.} \tag{2.1}$$

The sides of the component parallelepiped cells are necessarily the same as the sides of the complete figure.

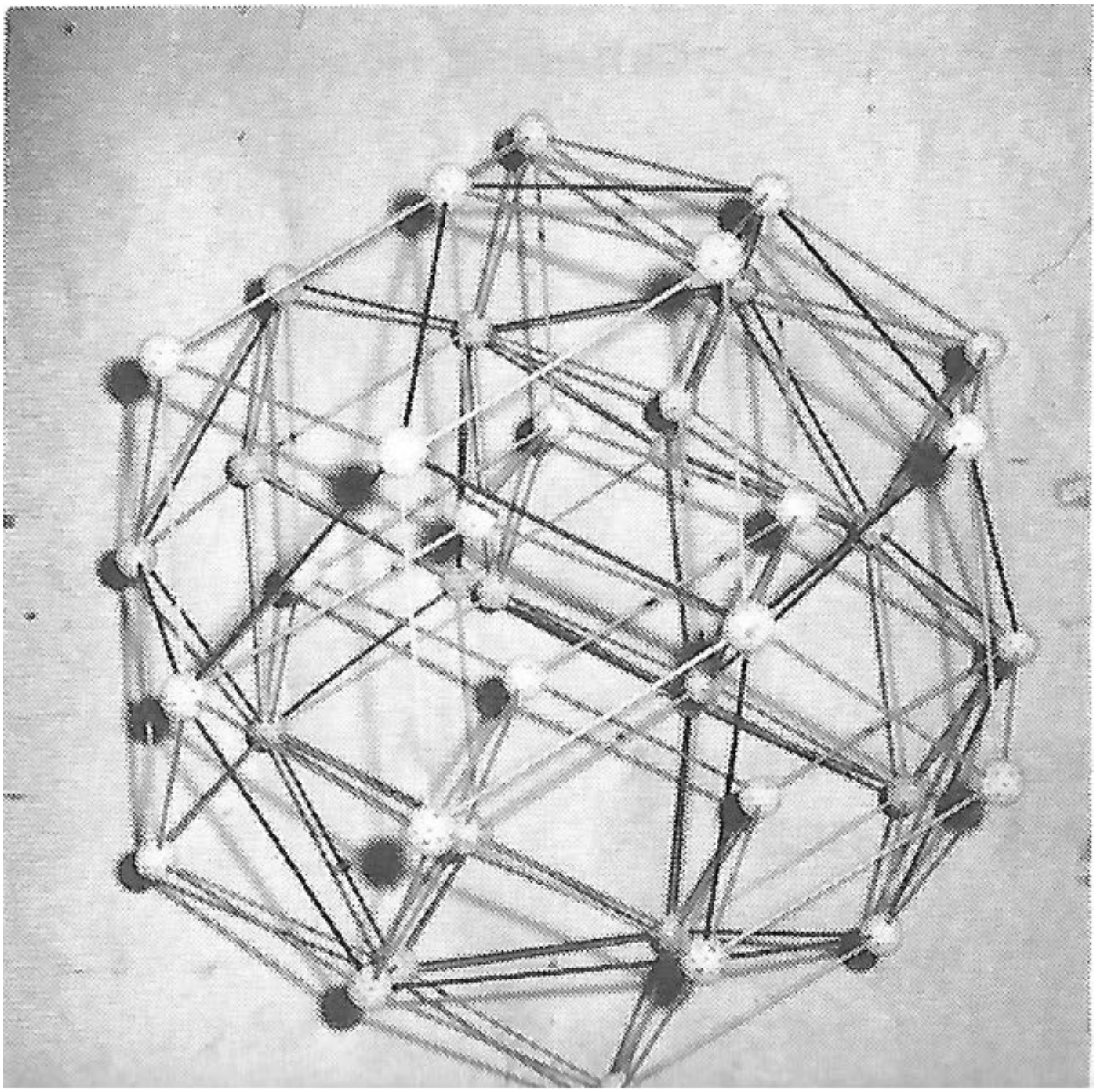


Figure 26: Triacontahedron

The triacontahedron subdivides into:

$$C_3^6 = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20 \text{ cells.} \tag{2.2}$$

The rhombic triacontahedron divides into 10 acute and 10 obtuse parallelepipeds.

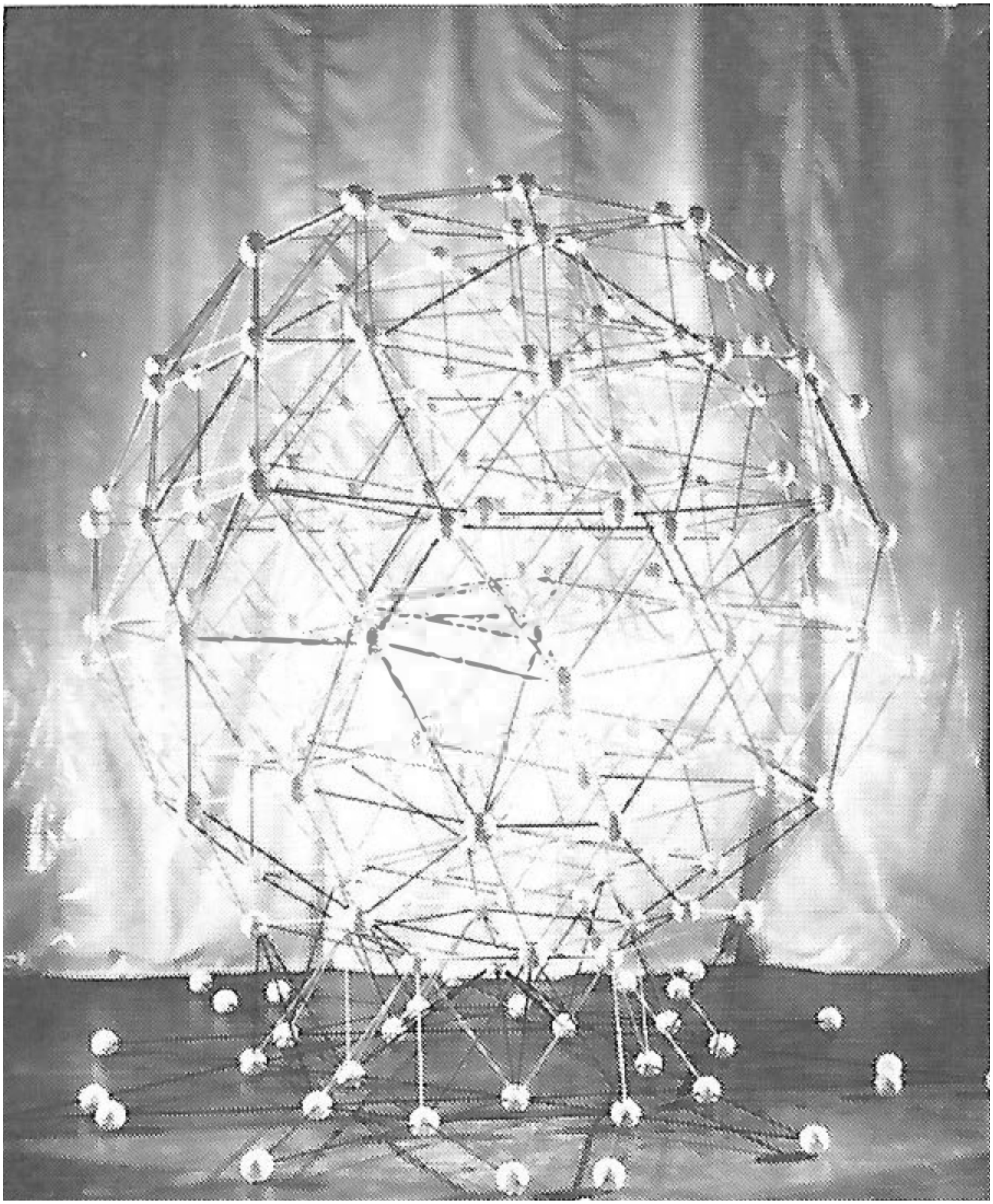


Figure 27: Enneacontahedron

The enneacontahedron subdivides into

$$C_3^{10} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120 \text{ cells.} \tag{2.3}$$

There are five different kinds of cells. With one kind of diamond, there are only two kinds of cells possible, but the enneacontahedron has two kinds of diamond faces allowing for more types of cells.

There are:

10	A cells	6 fat diamonds		acute
20	B cells	6 fat diamonds		obtuse
30	C cells	4 fat diamonds	2 skinny diamonds	acute
30	D cells	4 skinny diamonds	2 fat diamonds	acute
30	E cells	2 skinny diamonds	4 fat diamonds	obtuse
120	cells			

Table 2.3: Cells of the Enneacontahedron

3 Regular Ways of Arranging Lines in Space

The regular polyhedra are like seeds from which growths may appear. They are the connecting joints for the zonohedra. The joints are all parallel to each other. The lines of the zonohedra are perpendicular to the faces of the joints.

3.1 Three Zones

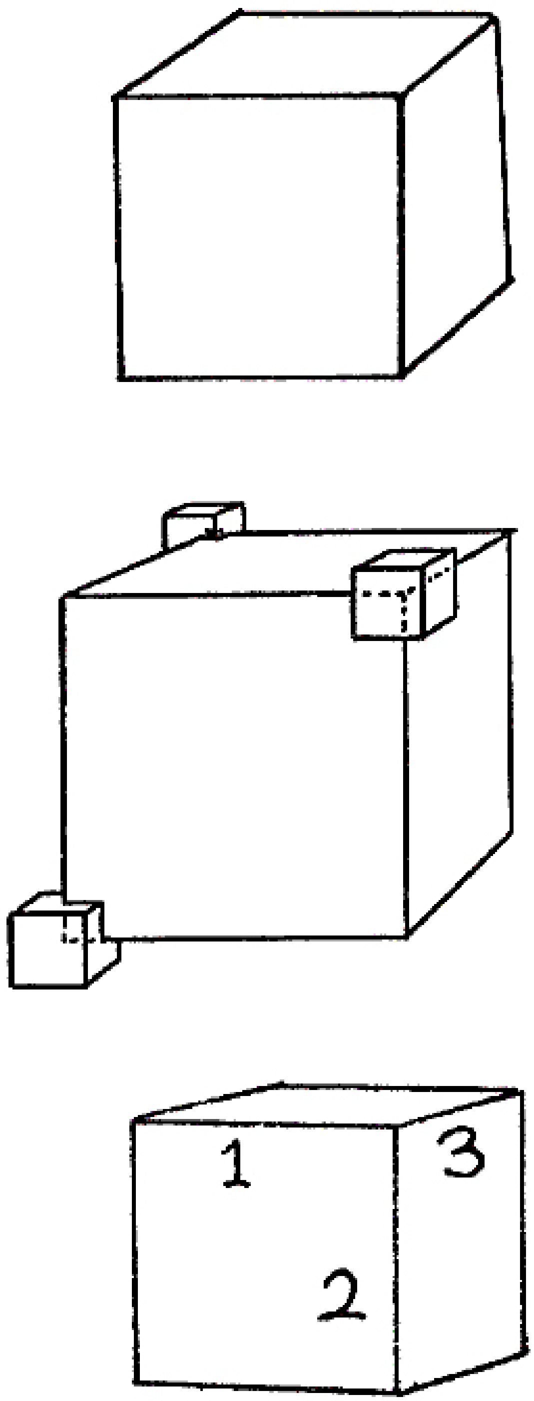


Figure 28: Cube

3.2 Four Zones

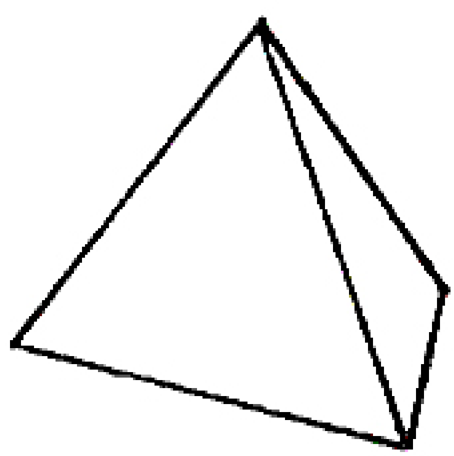


Figure 29: Tetrahedron

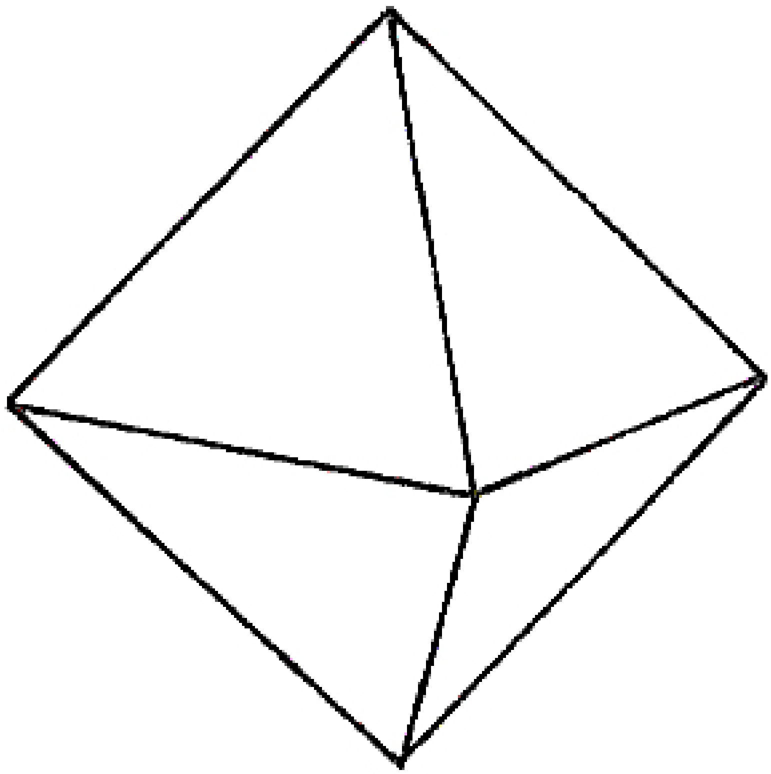


Figure 30: Octahedron

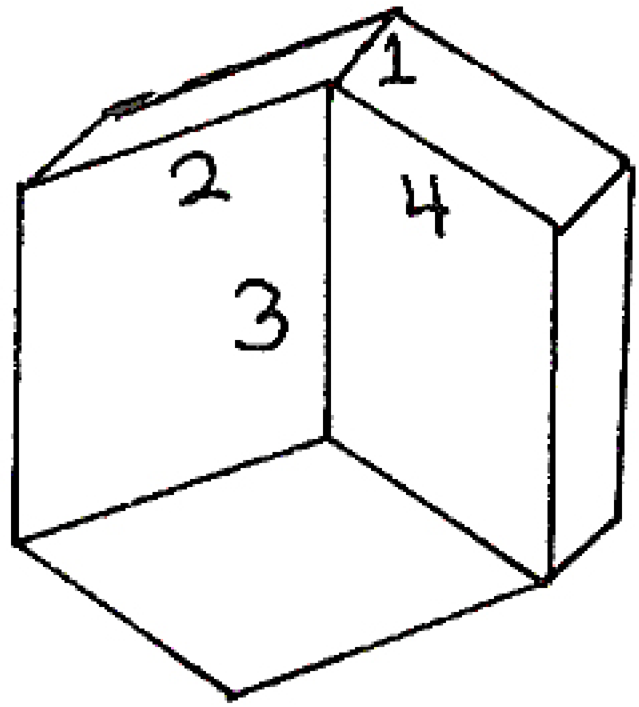


Figure 31: Rhombic Dodecahedron

3.3 Six Zones

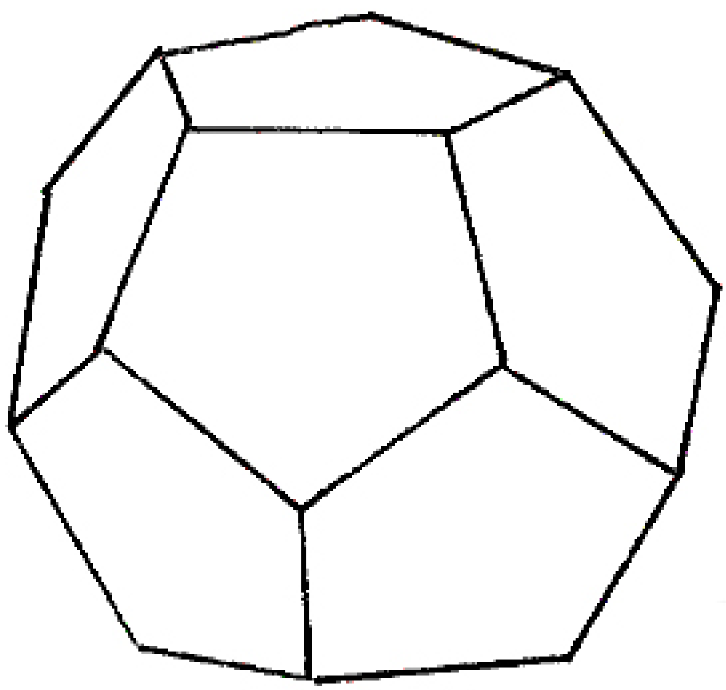


Figure 32: Dodecahedron

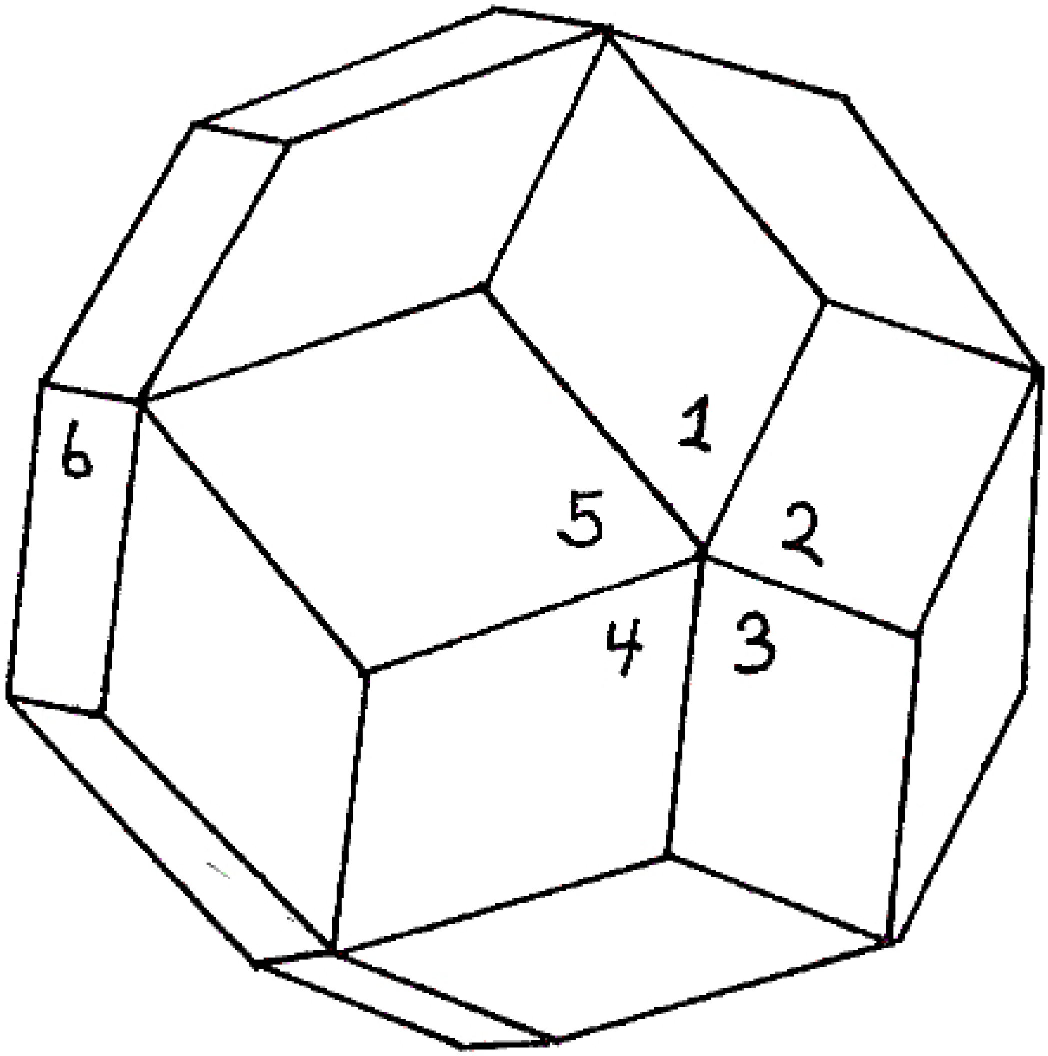


Figure 33: Rhombic Triacontahedron

3.4 Ten Zones

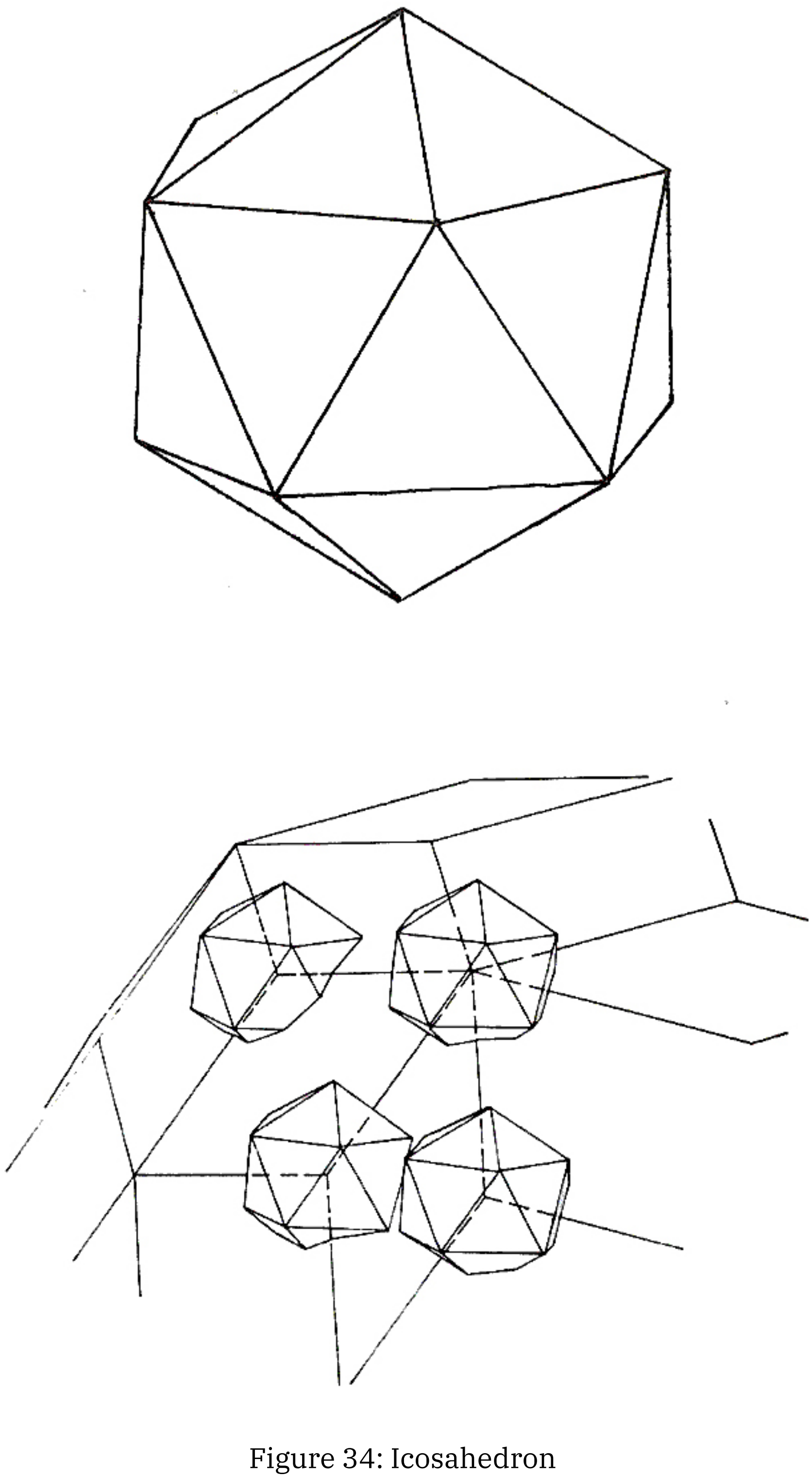


Figure 34: Icosahedron

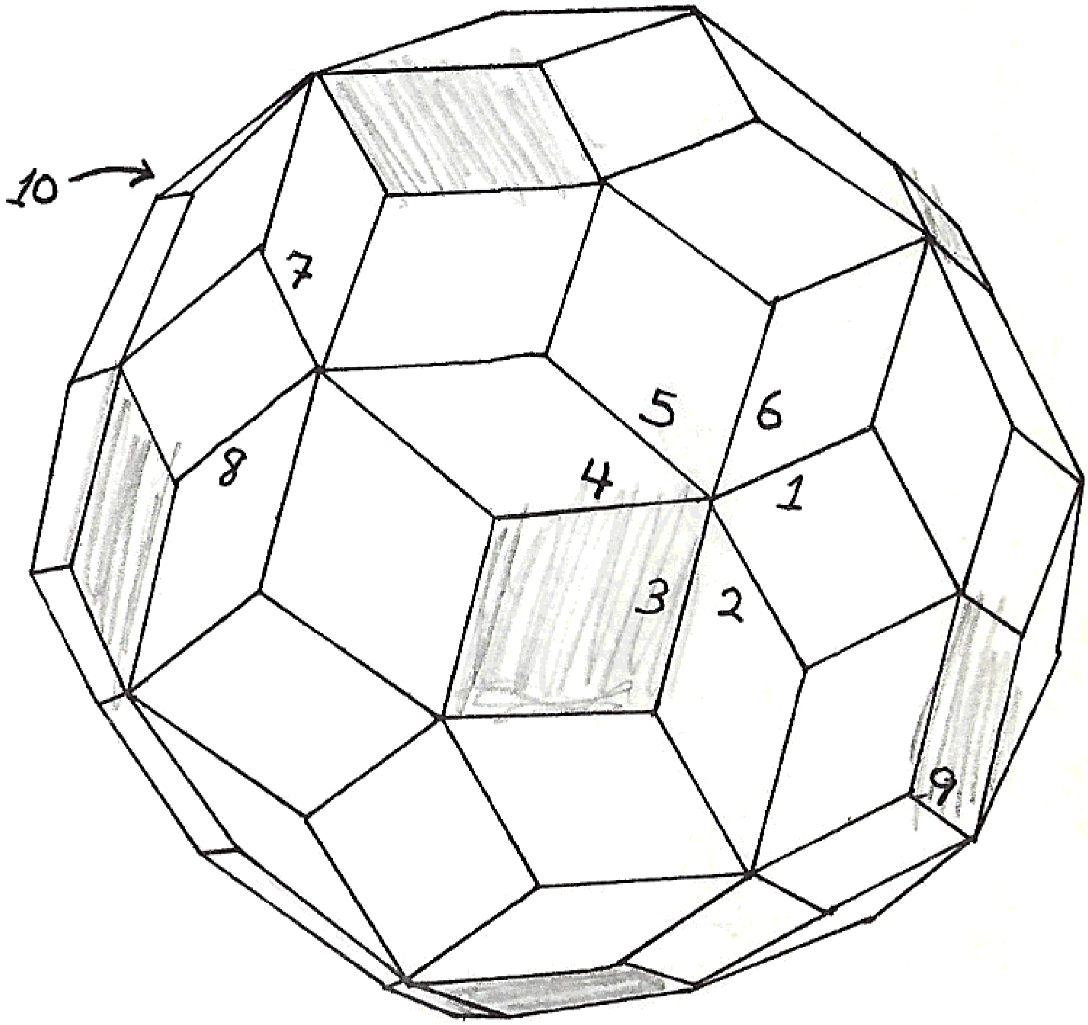


Figure 35: Rhombic Enneacontahedron

4 Clustering

4.1 Twenty-one Zone Structure

Triacontahedron

In this series, the central zome is higher. It could as well be lower.

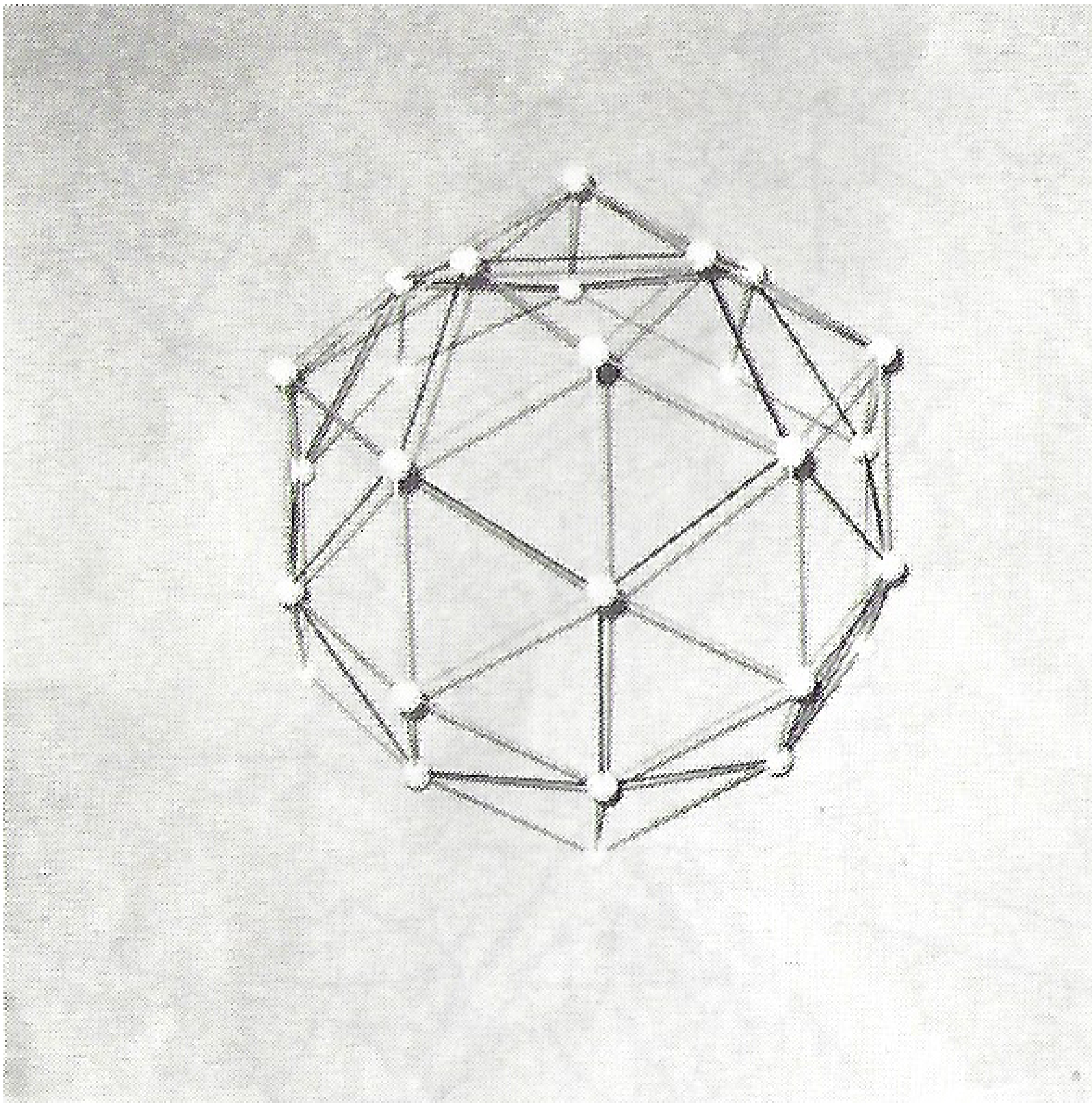


Figure 36: Triacontahedron, 1

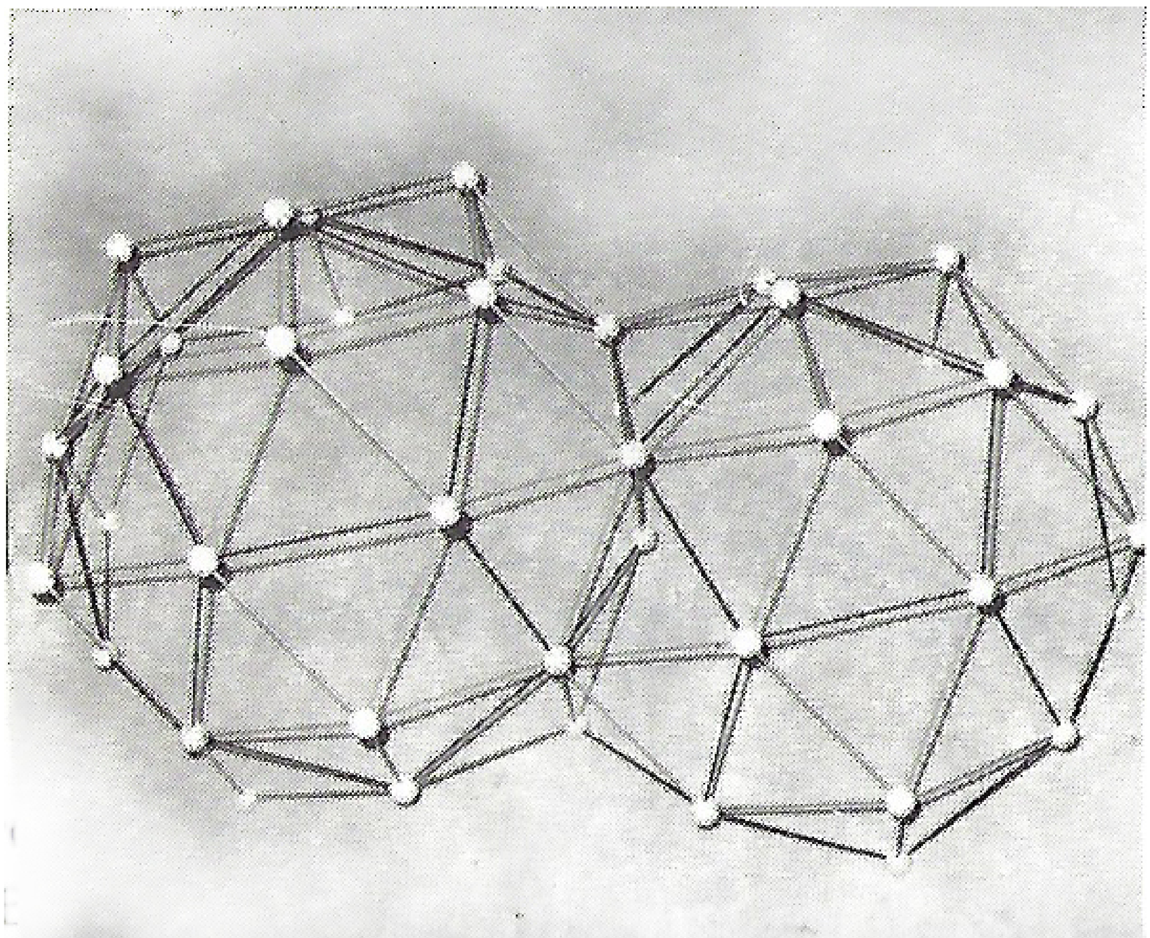


Figure 37: Triacontahedron, 2

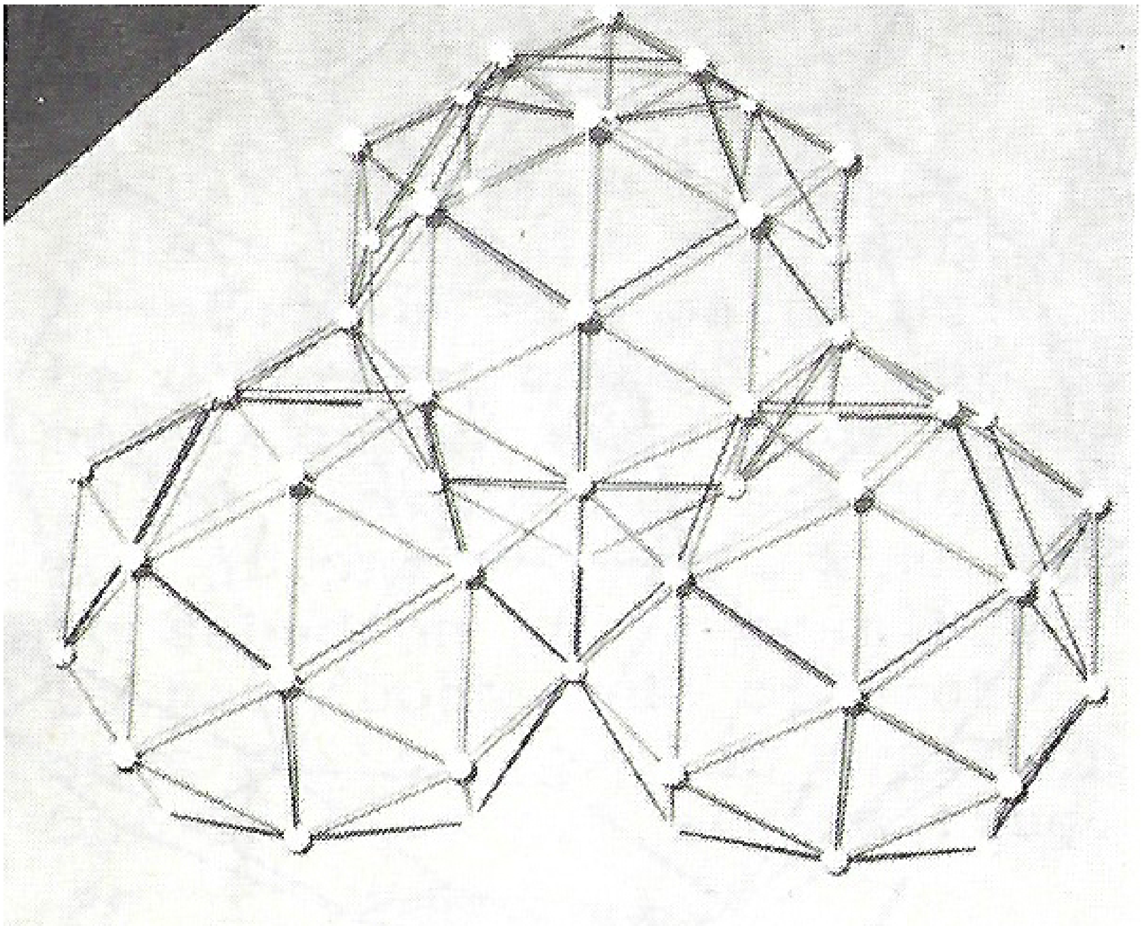


Figure 38: triacontahedron, 3

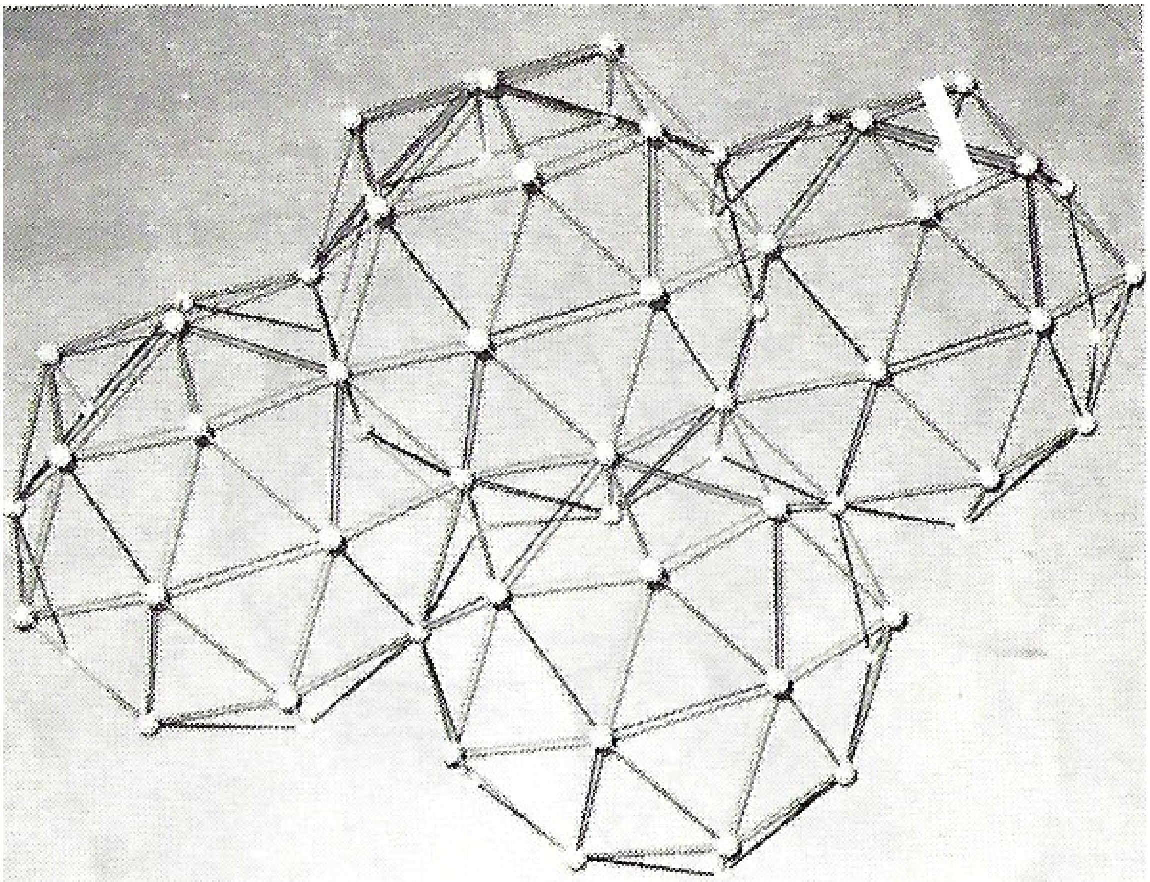


Figure 39: triacontahedron, 4

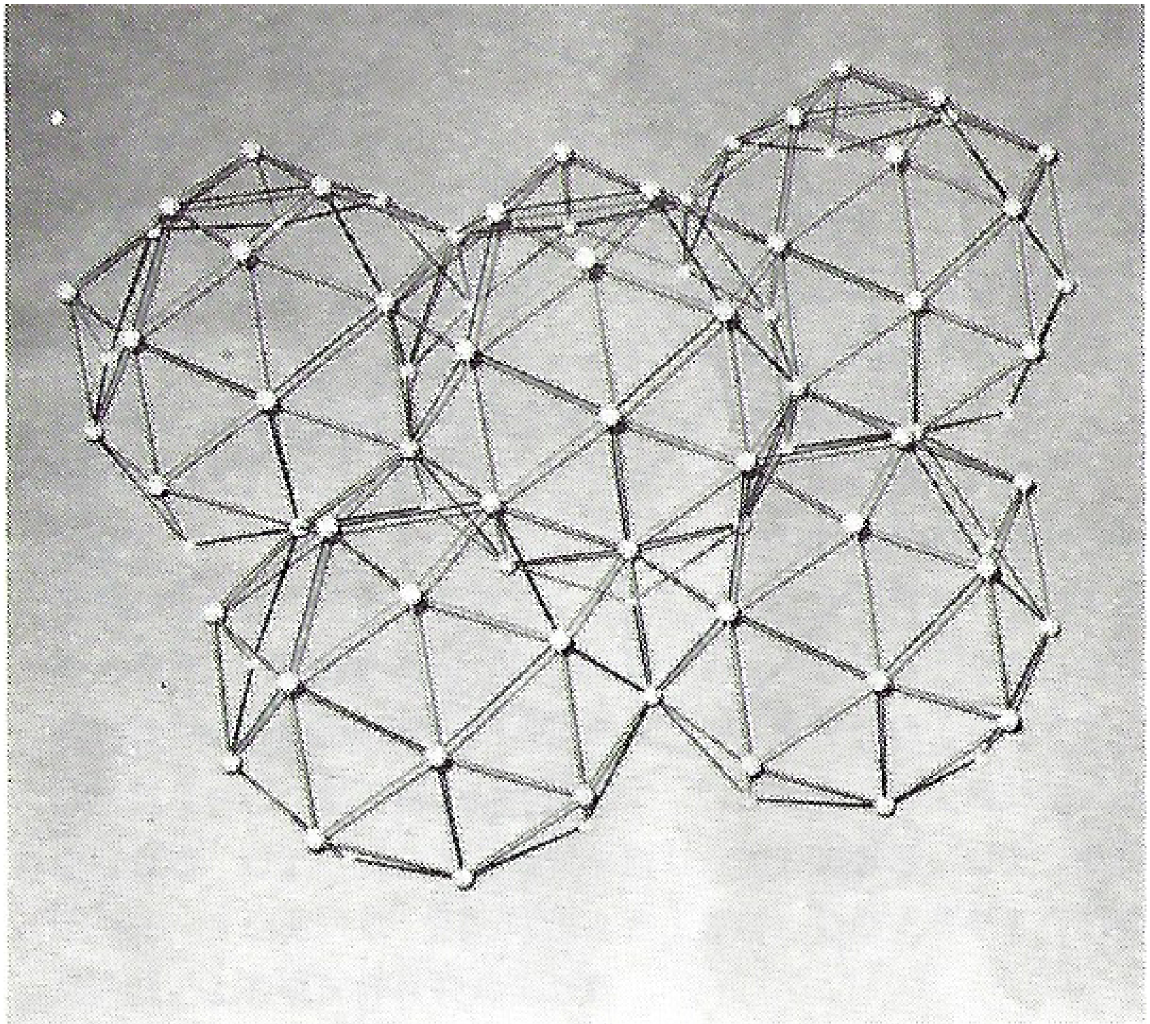


Figure 40: triacontahedron, 5

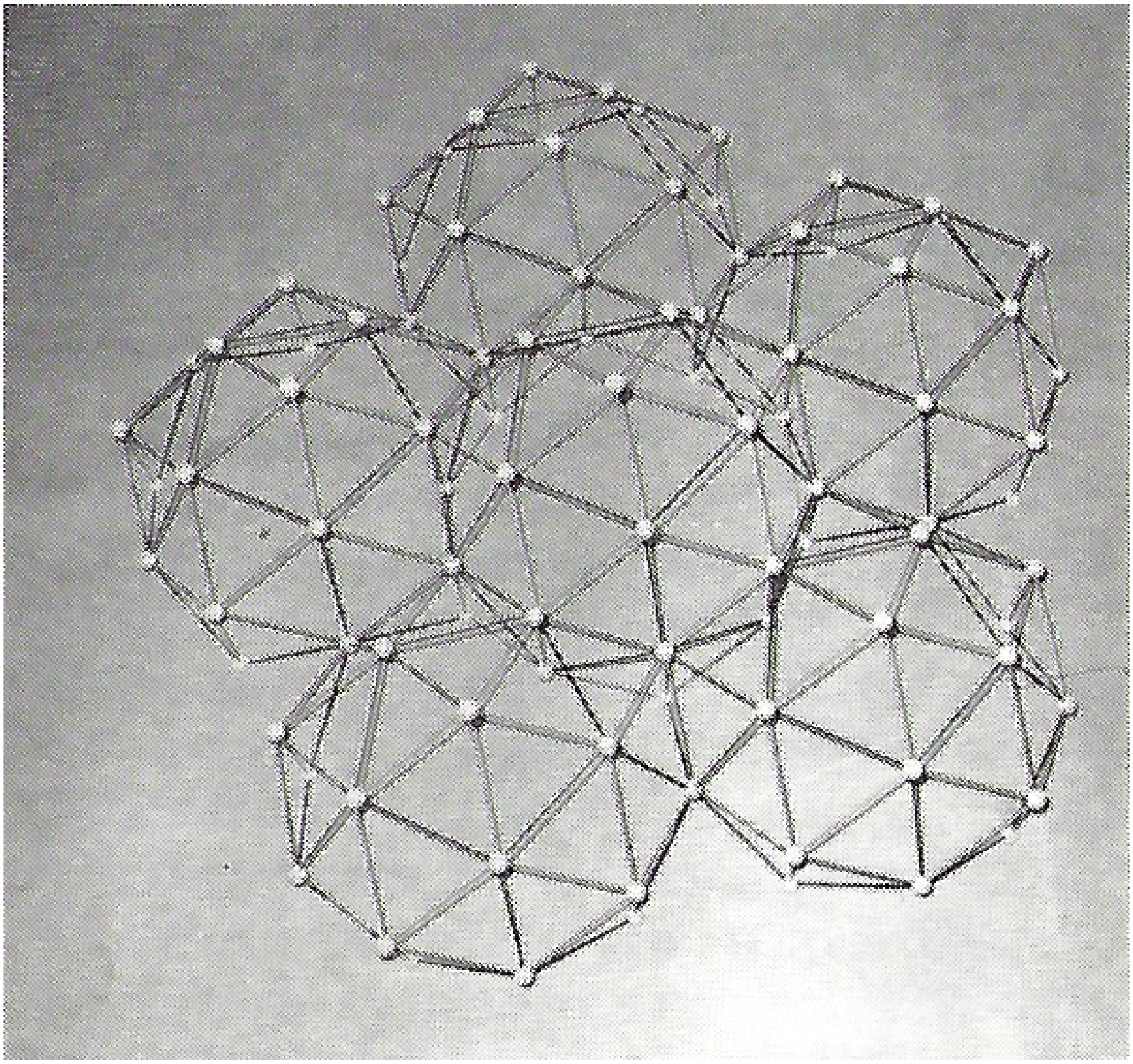


Figure 41: triacontahedron, 5

Enneacontahedron of edge length B clustered with triacontahedron of edge length A and triacontahedron of edge length AT^{-1}.

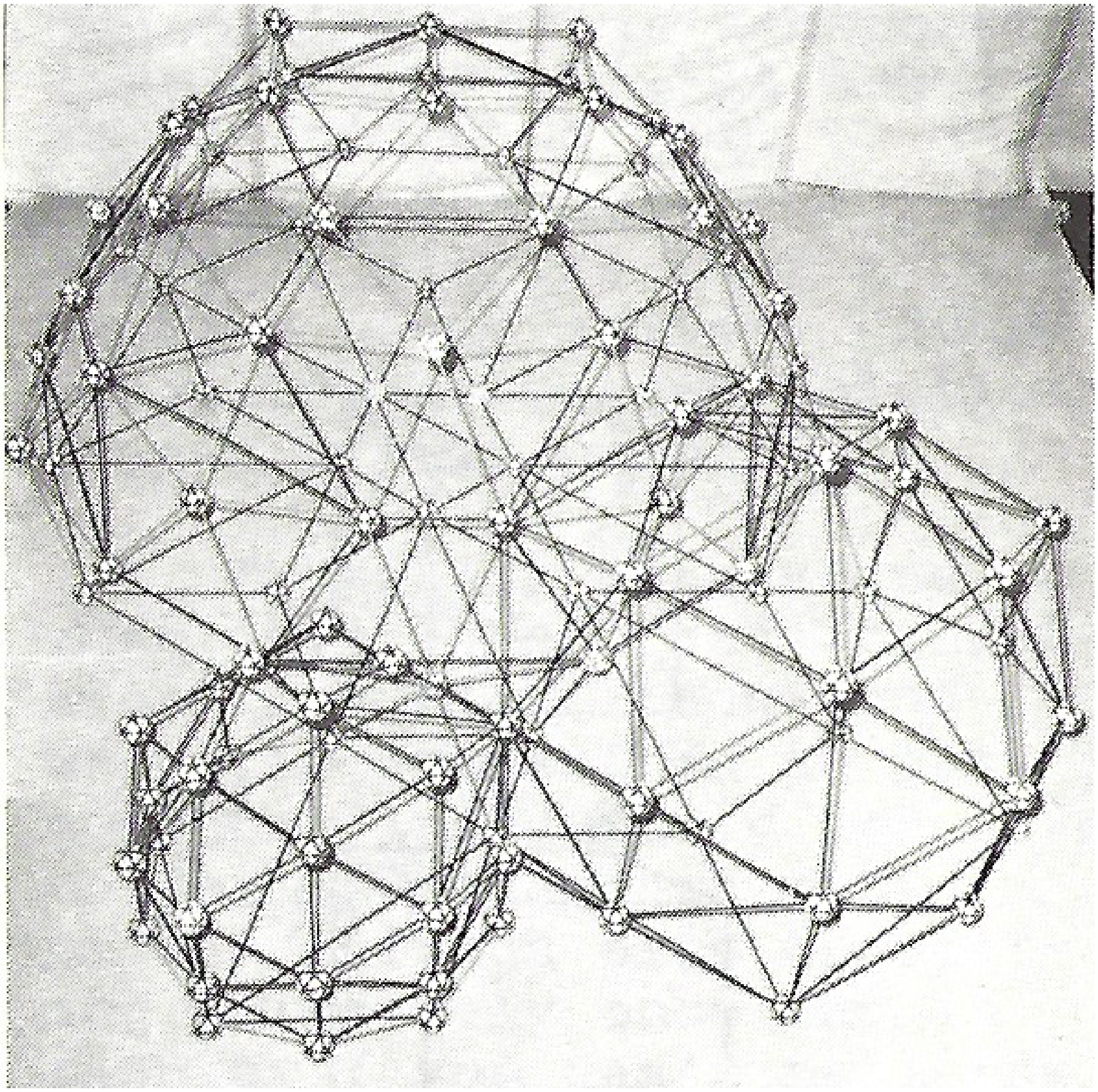


Figure 42: Enneacontahedron with 2 triacontahedrons

Clusters of triacontahedra - A line vertical.

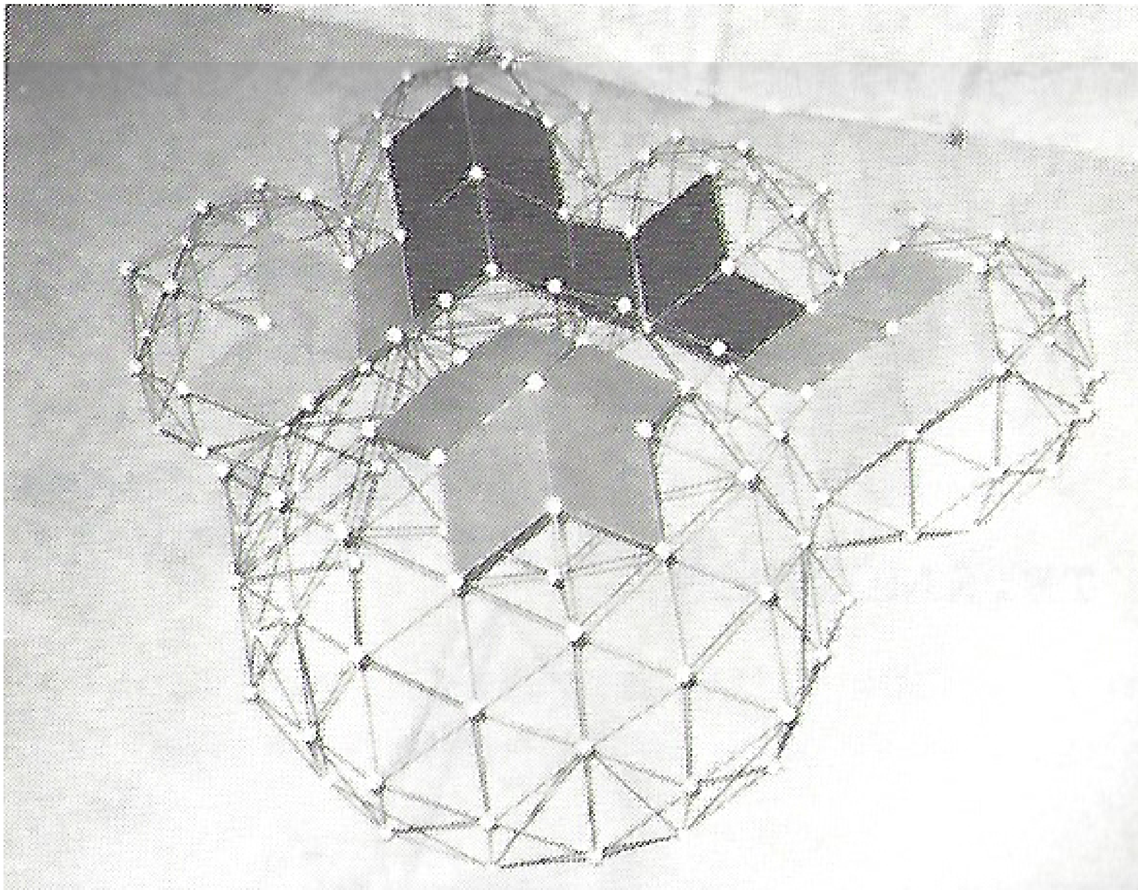


Figure 43: Triacontahedra, 5

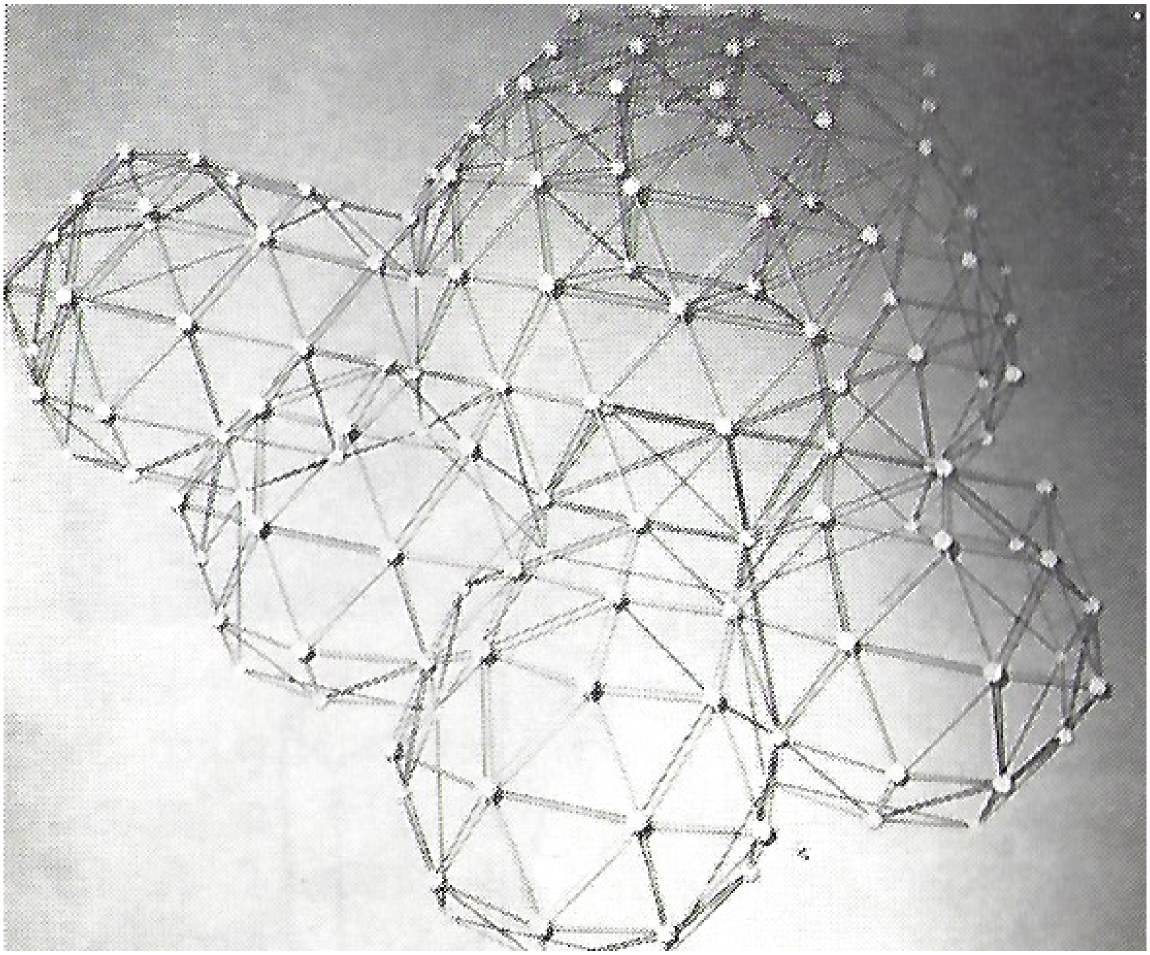


Figure 44: triacontahedra, 5

4.2 Twenty-one Zone Zome Clusters

6 A LINES 15 C LINES Orientation: A lines horizontal

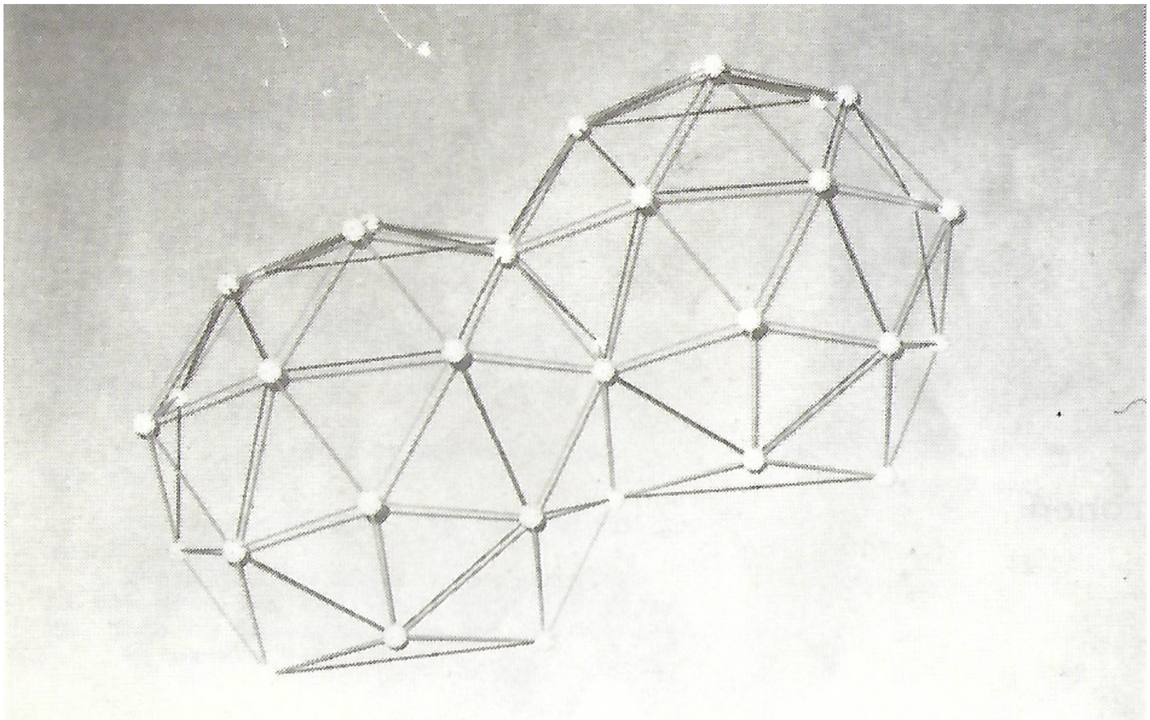


Figure 45: Two triacontahedra fused through skew hexagon.

Two triacontahedra fused through skew hexagon.

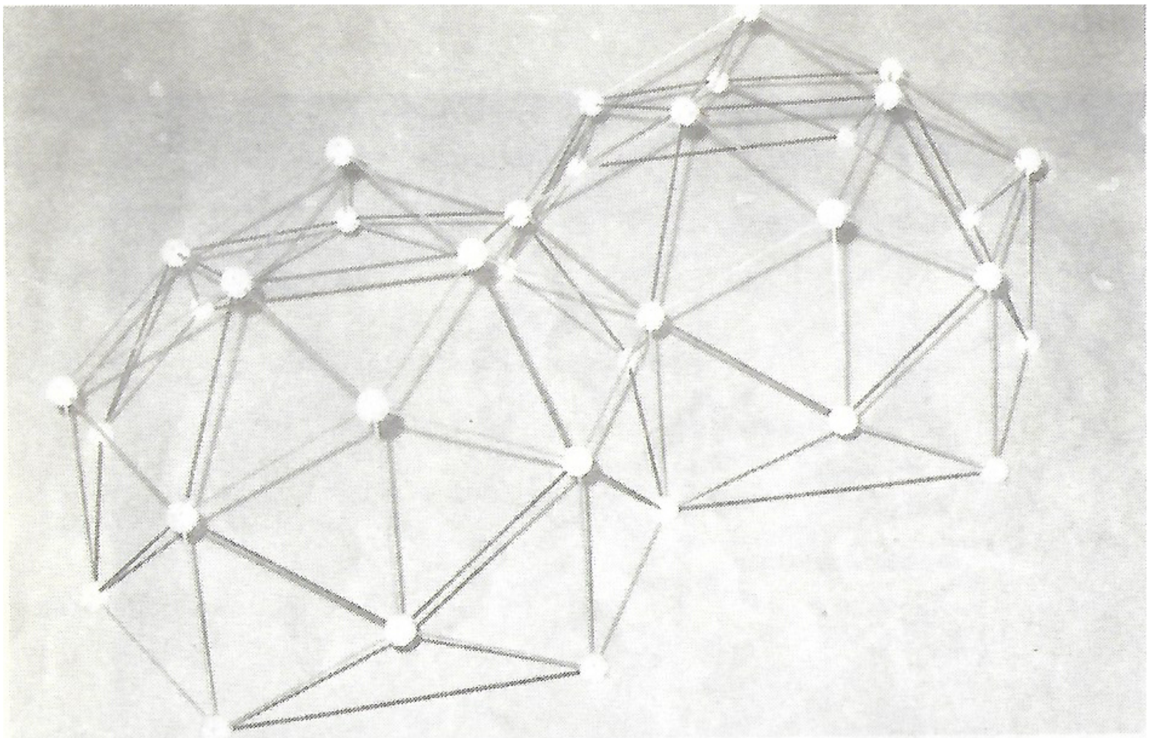


Figure 46: Front view - same as above.

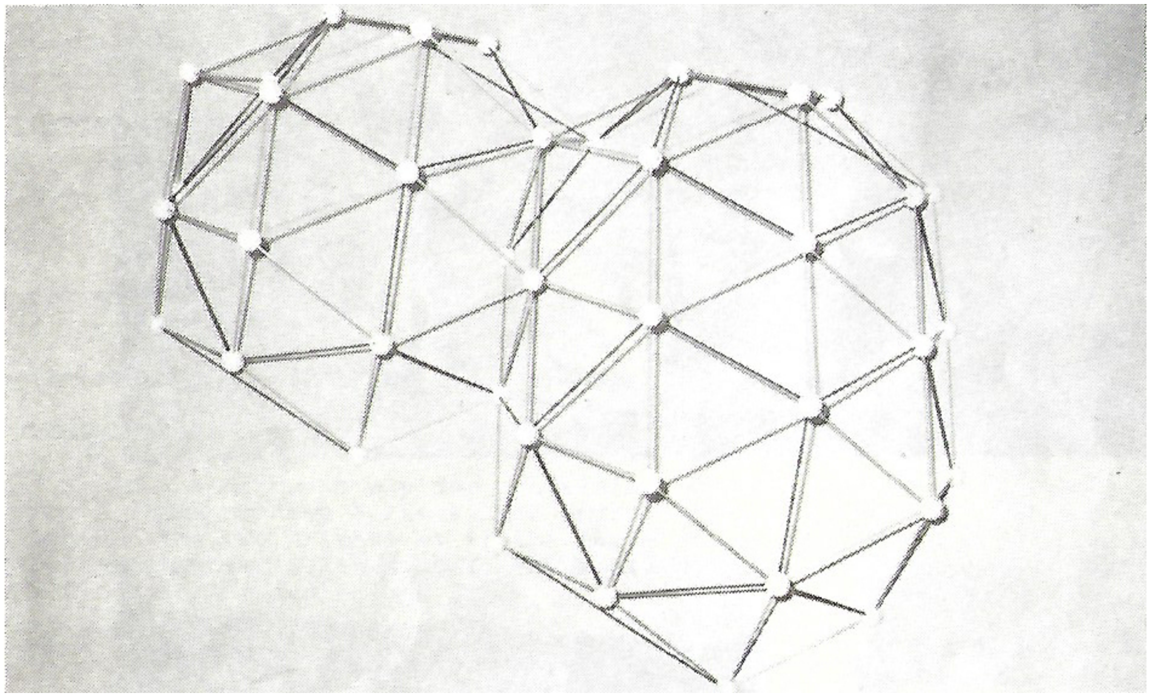


Figure 47: Same cluster as above - One zome has zone stretched past others.

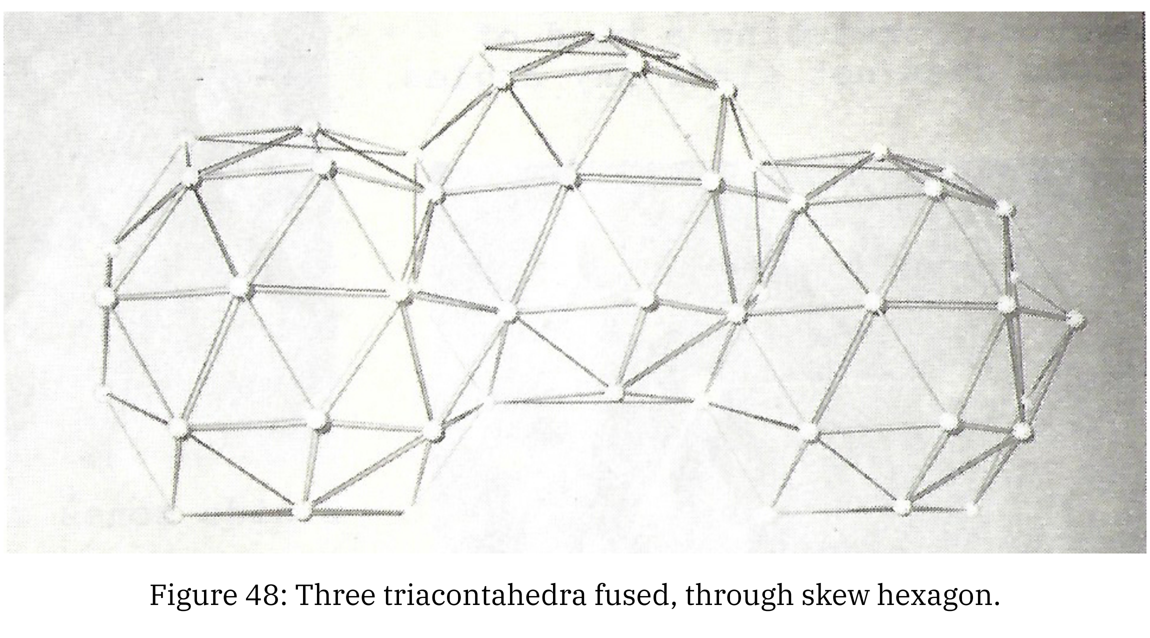


Figure 48: Three triacontahedra fused, through skew hexagon.

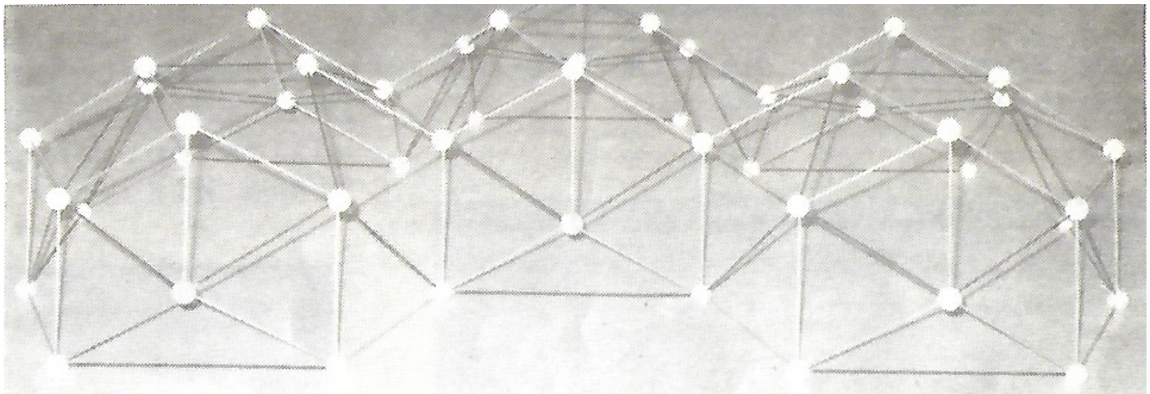


Figure 49: Front view - same as above.

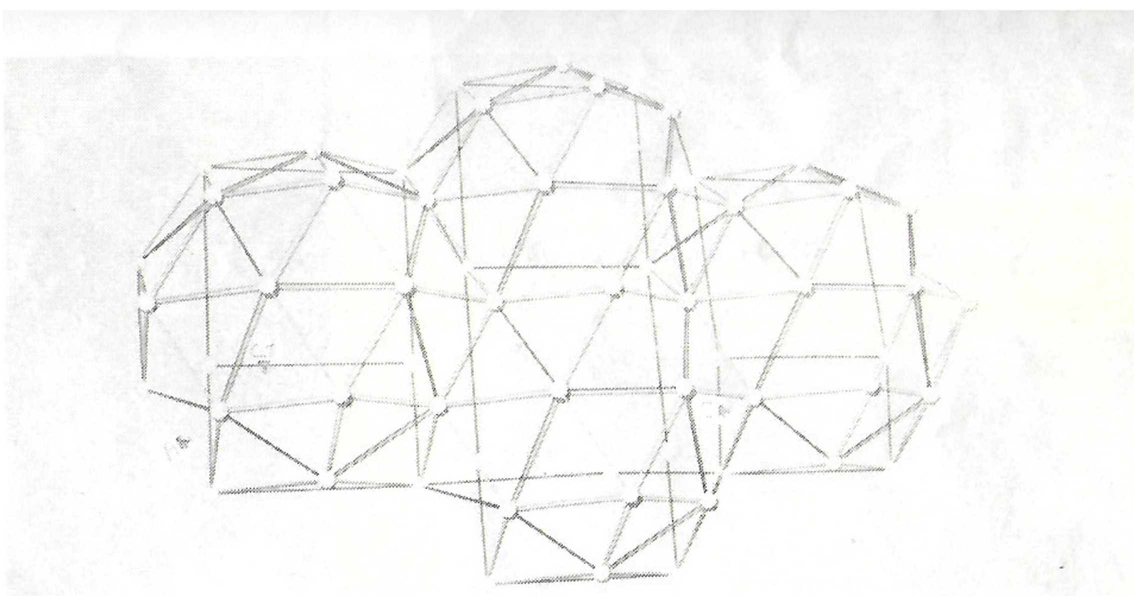


Figure 50: Fourth zome added - fused to two side zomes through skew hexagon - fused to front zome through vertical section.

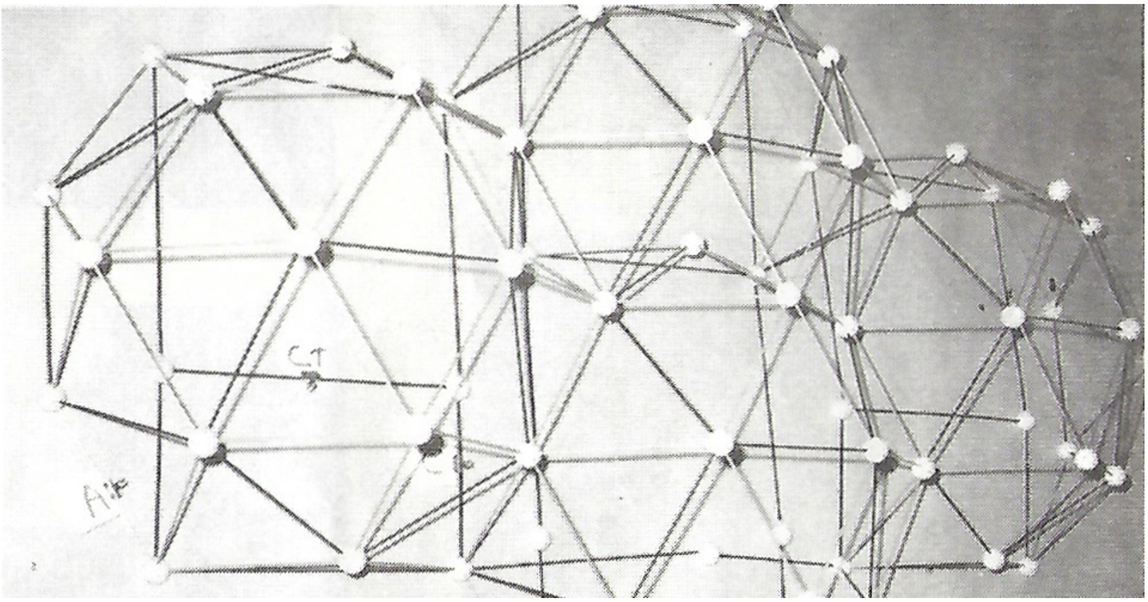


Figure 51: Same as above. Notice the appearance of C and CT in the floor sections.

5 Stretching a Zone

Zonohedra have bands of parallel edges. Any such band of edges can be stretched to alter the shape of the zonohedron. Stretching a band of edges does not alter any angles.

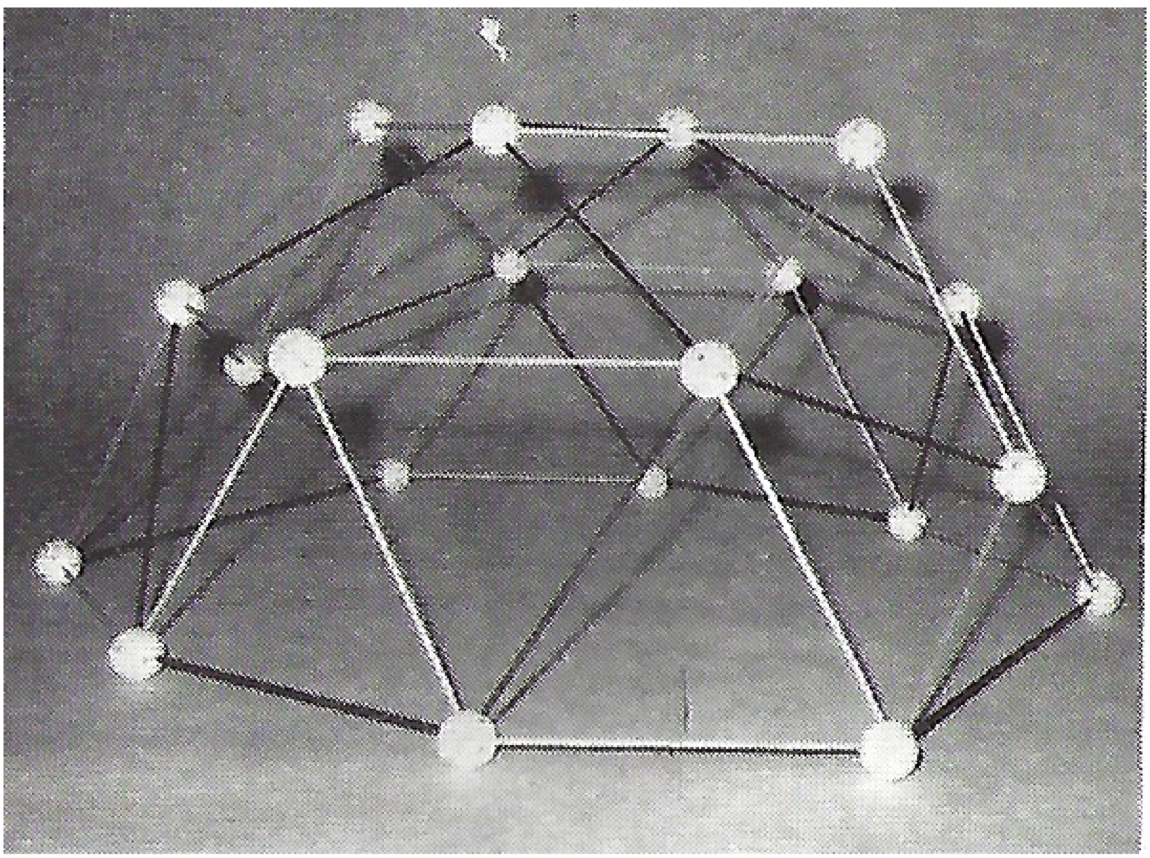


Figure 52: Six zone zome - half a triacontahedron

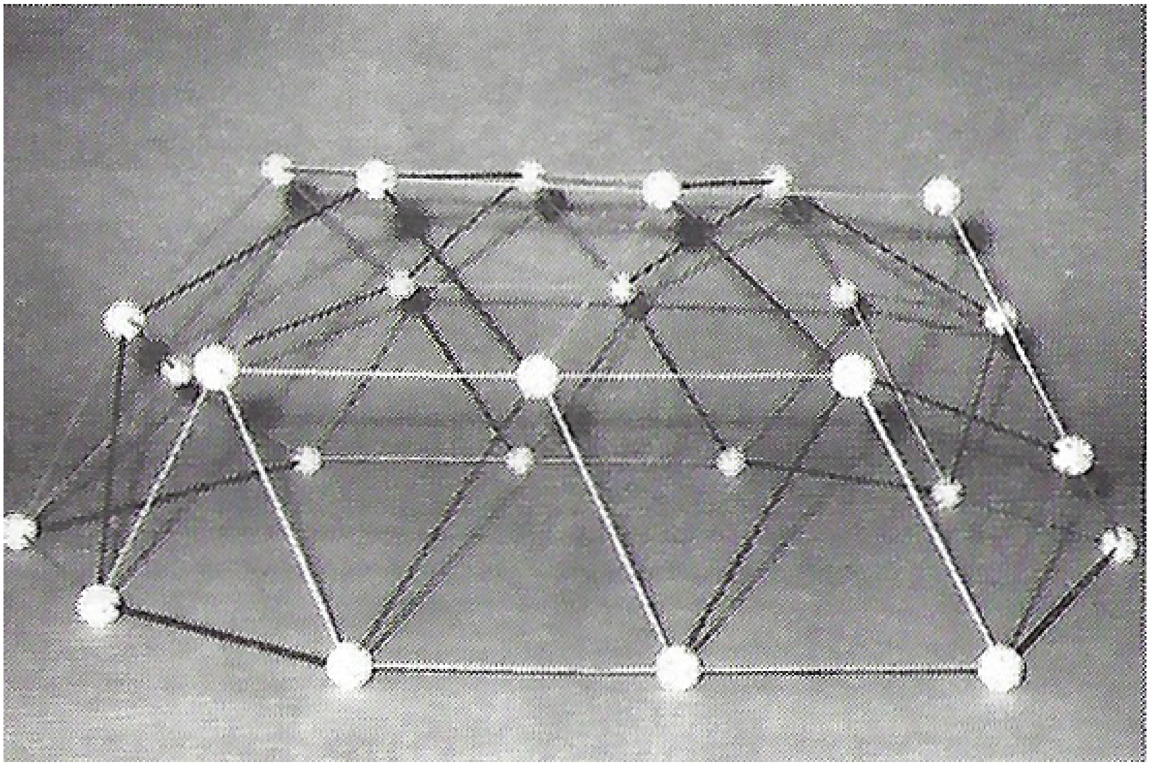


Figure 53: 1 horizontal zone stretched to 2

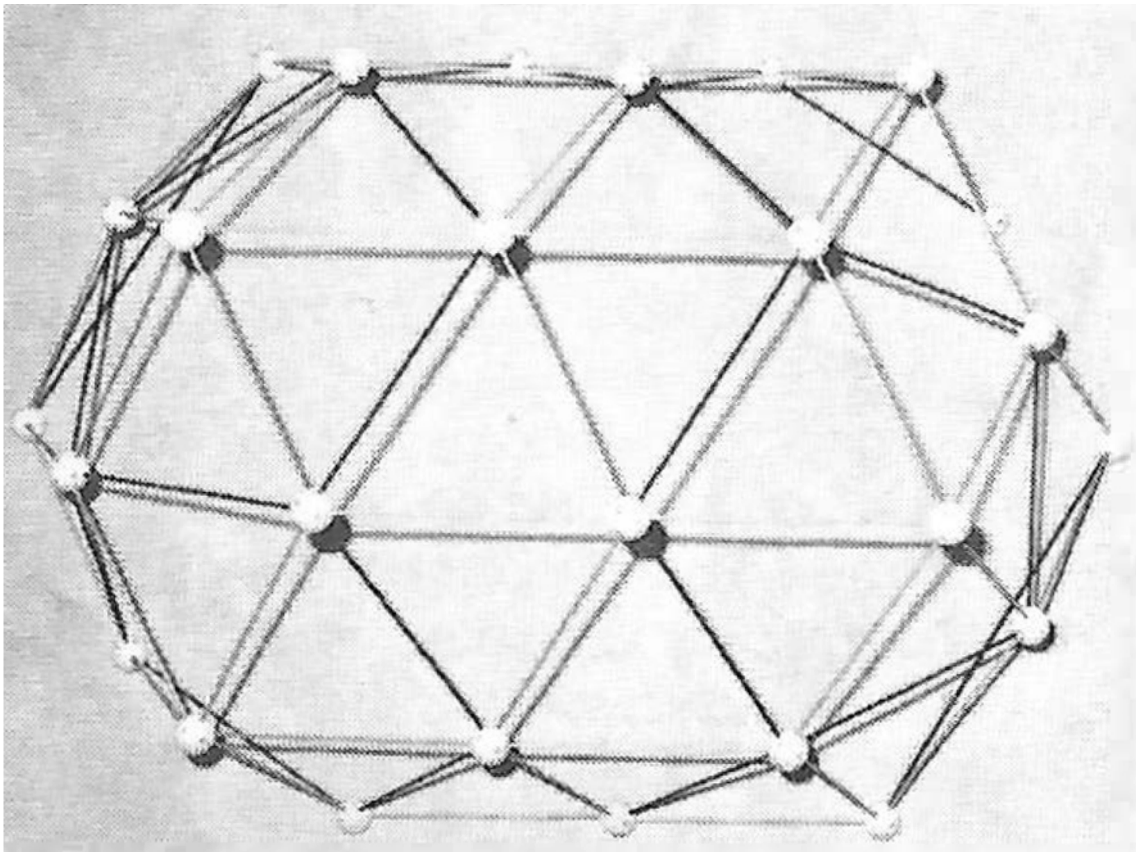


Figure 54: same as above (top view)

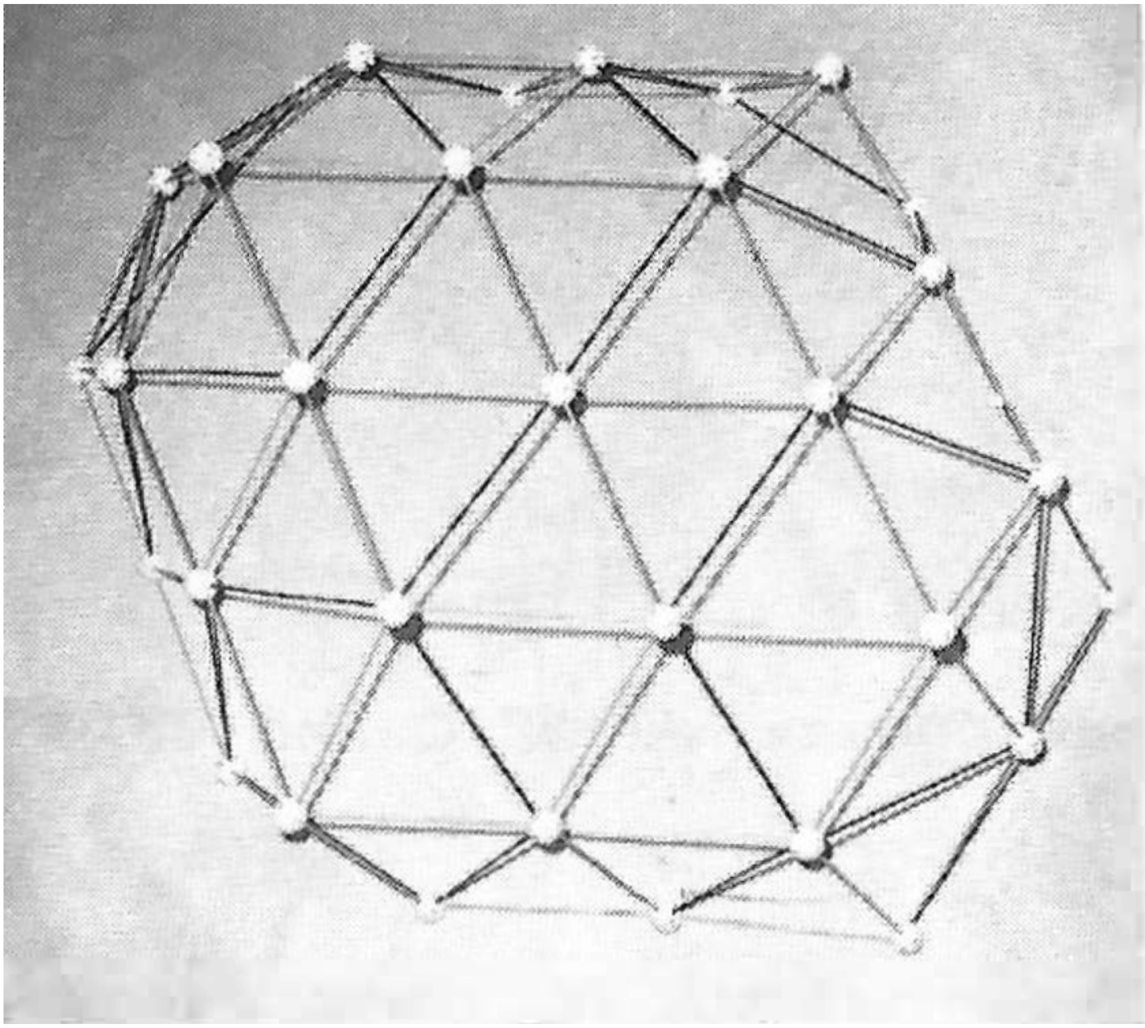


Figure 55: 2nd horizontal zone stretched to two top panels 2x2 (top view)

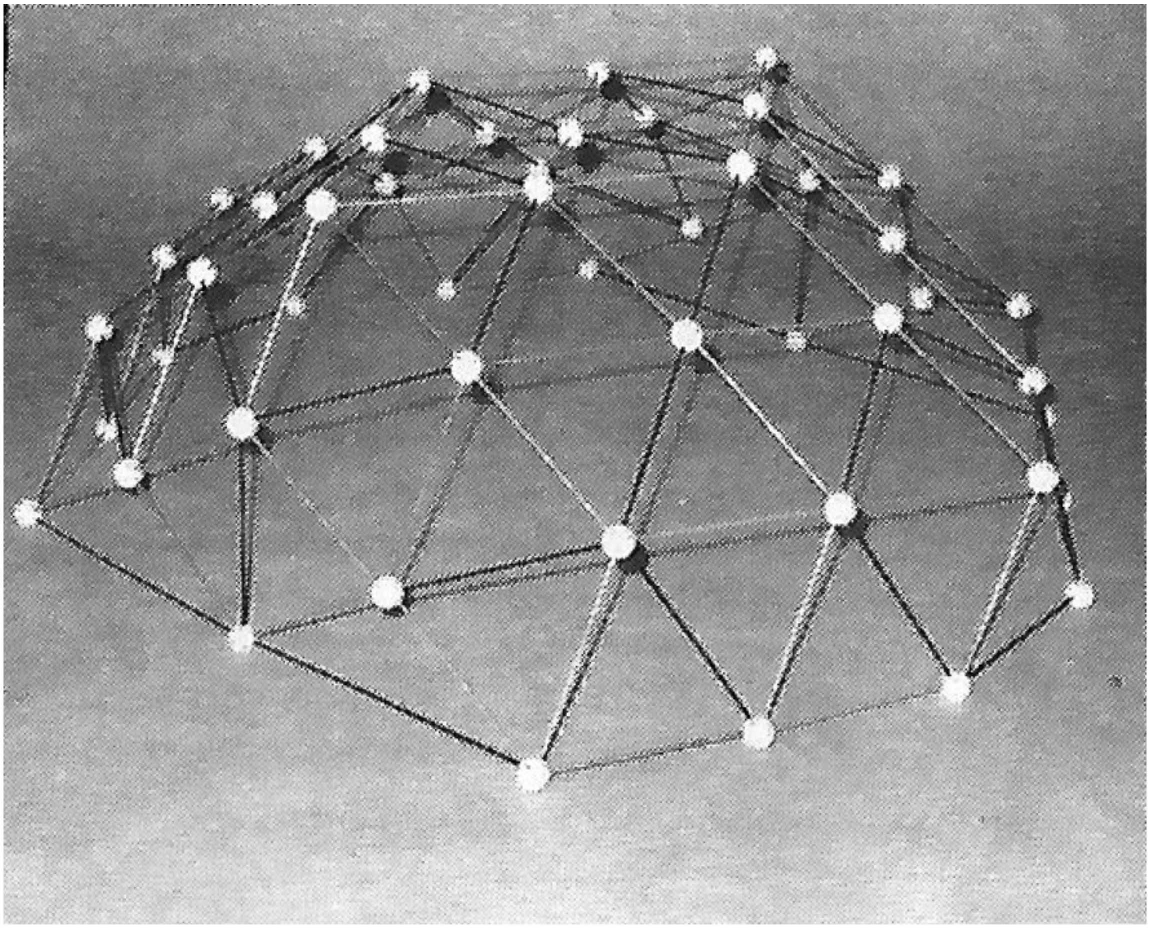


Figure 56: 2 side zones stretched

Stretching zones allows one to build buildings of different shapes using the same kinds of components.

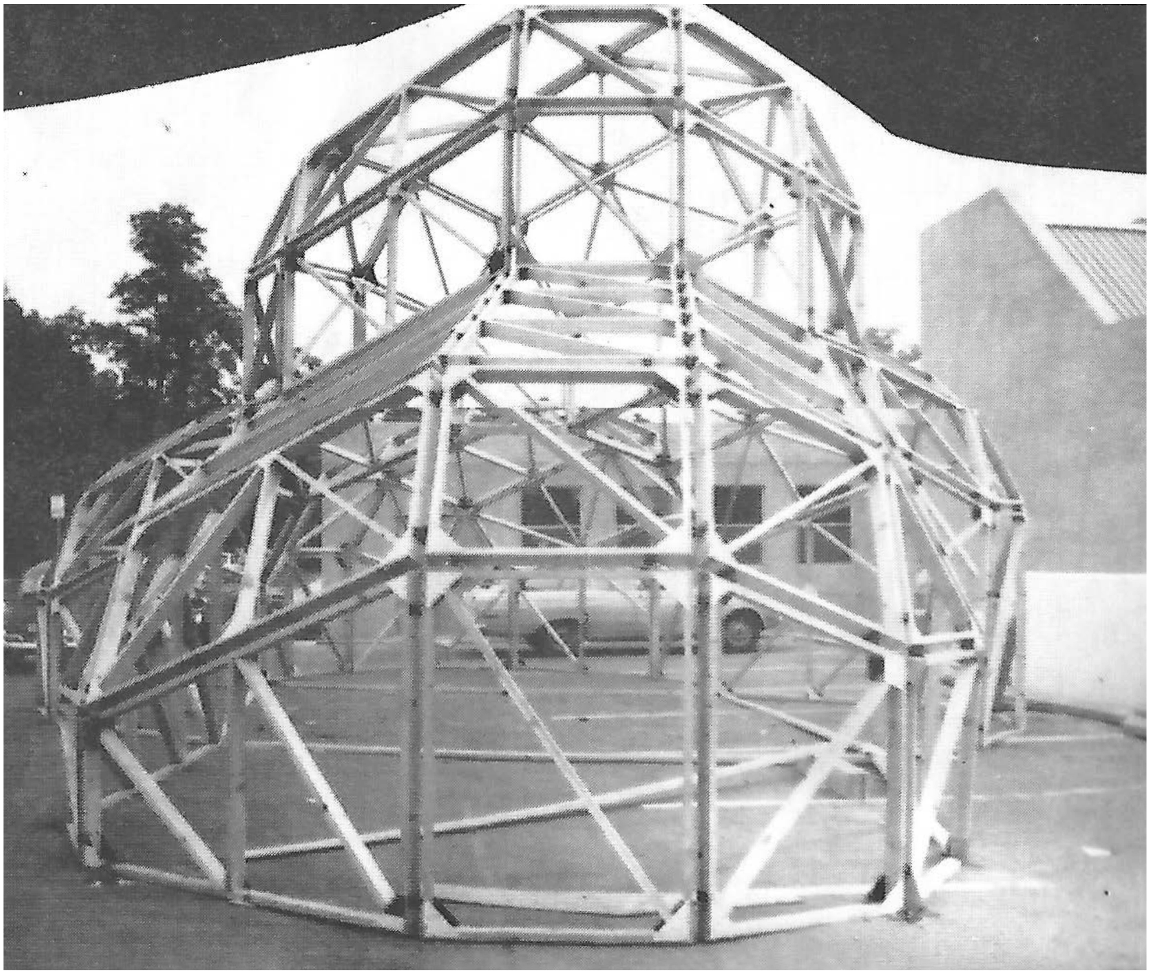


Figure 57

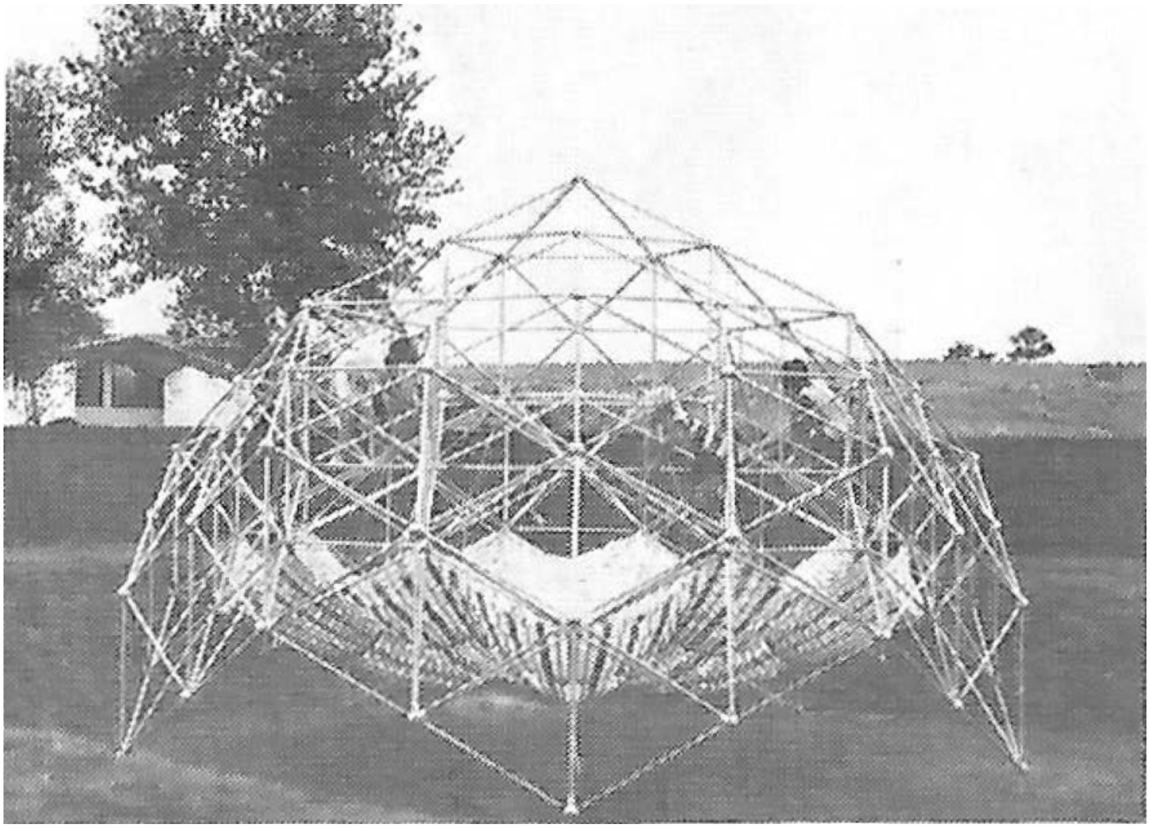


Figure 58

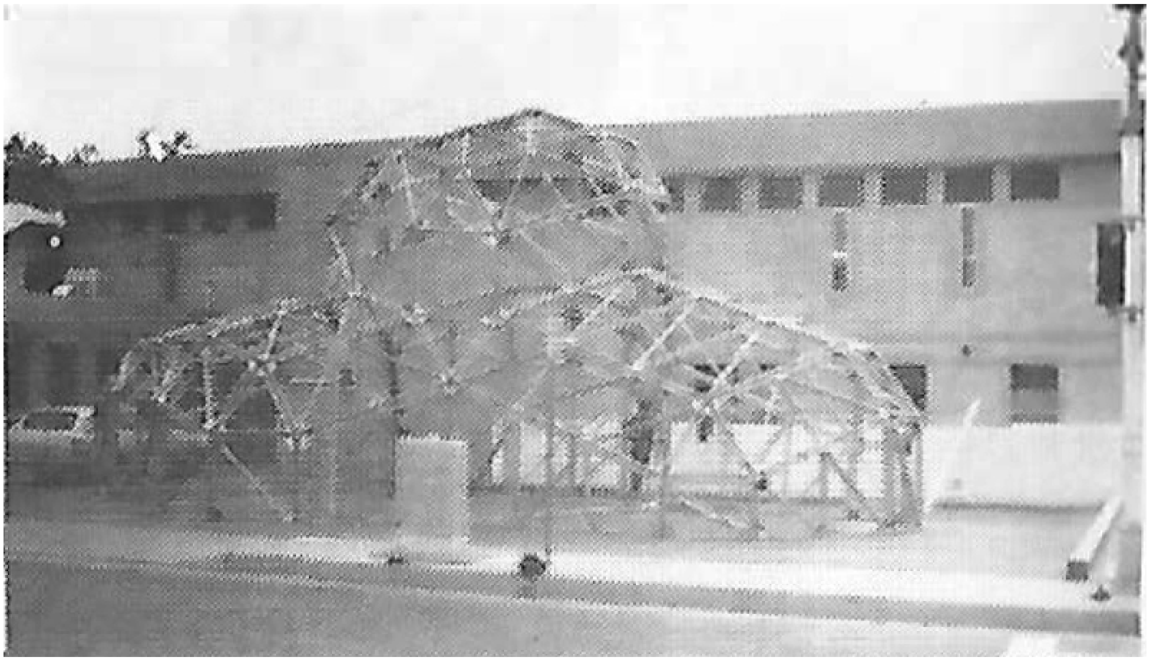


Figure 59

6 Trusses: Twenty-one Zone System

6.1 6 A Lines 15 C Lines

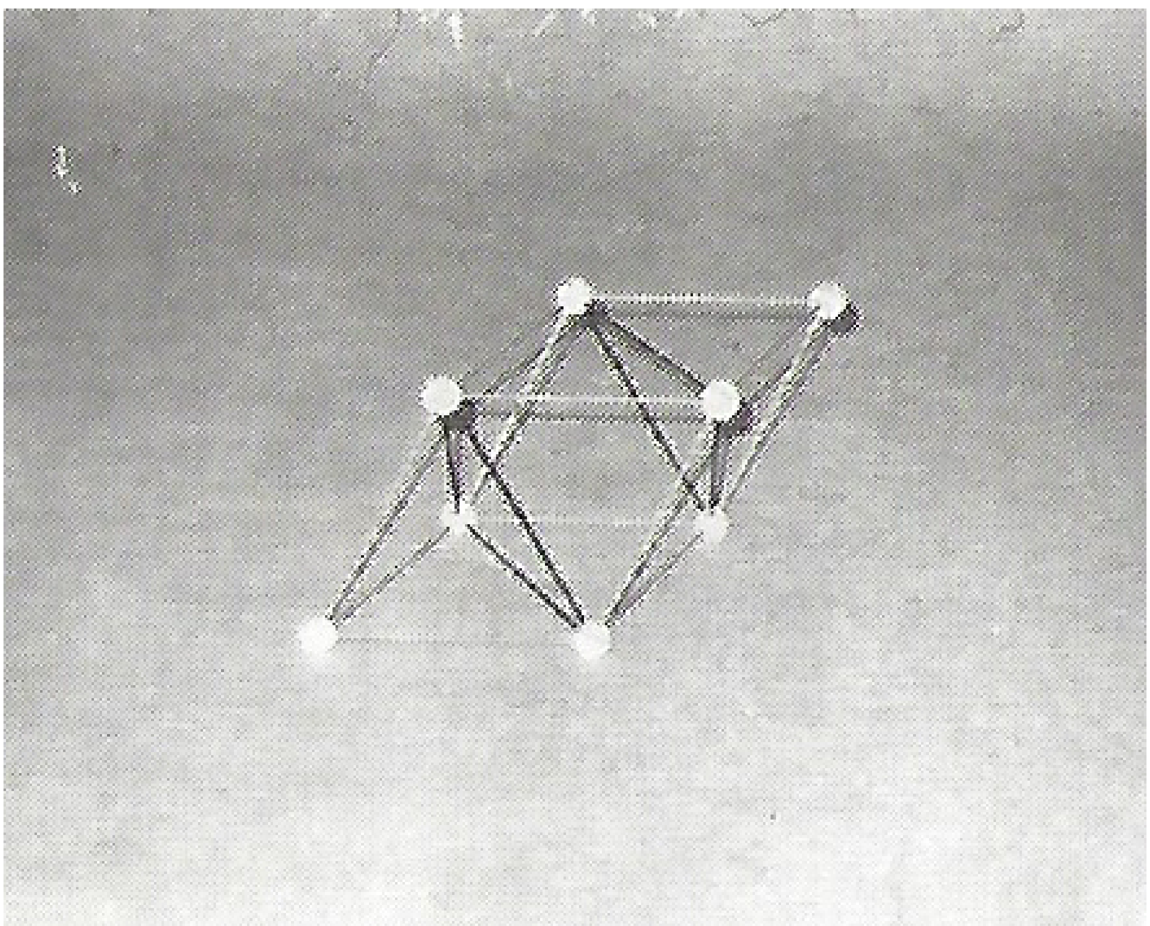


Figure 60: Single 21-zone A cell

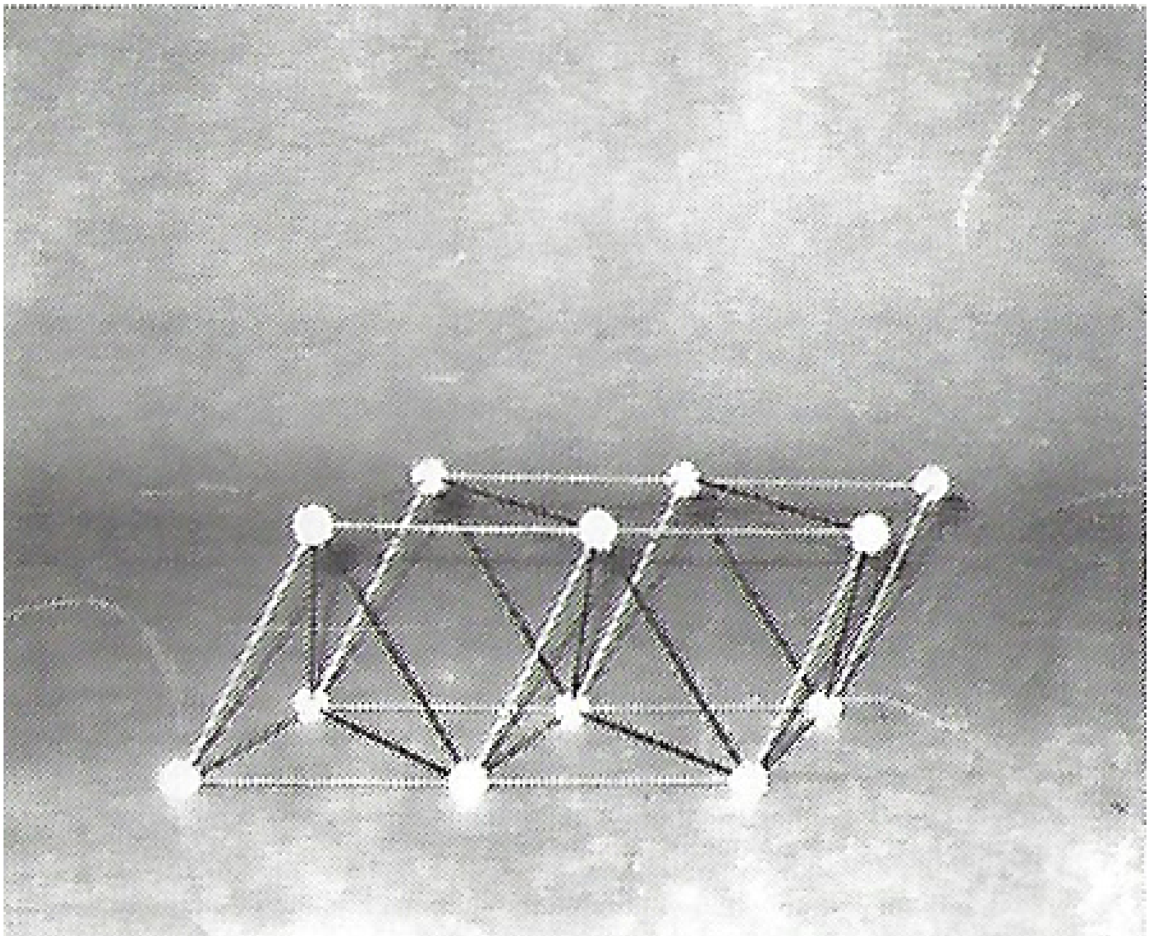


Figure 61: 2 A cells

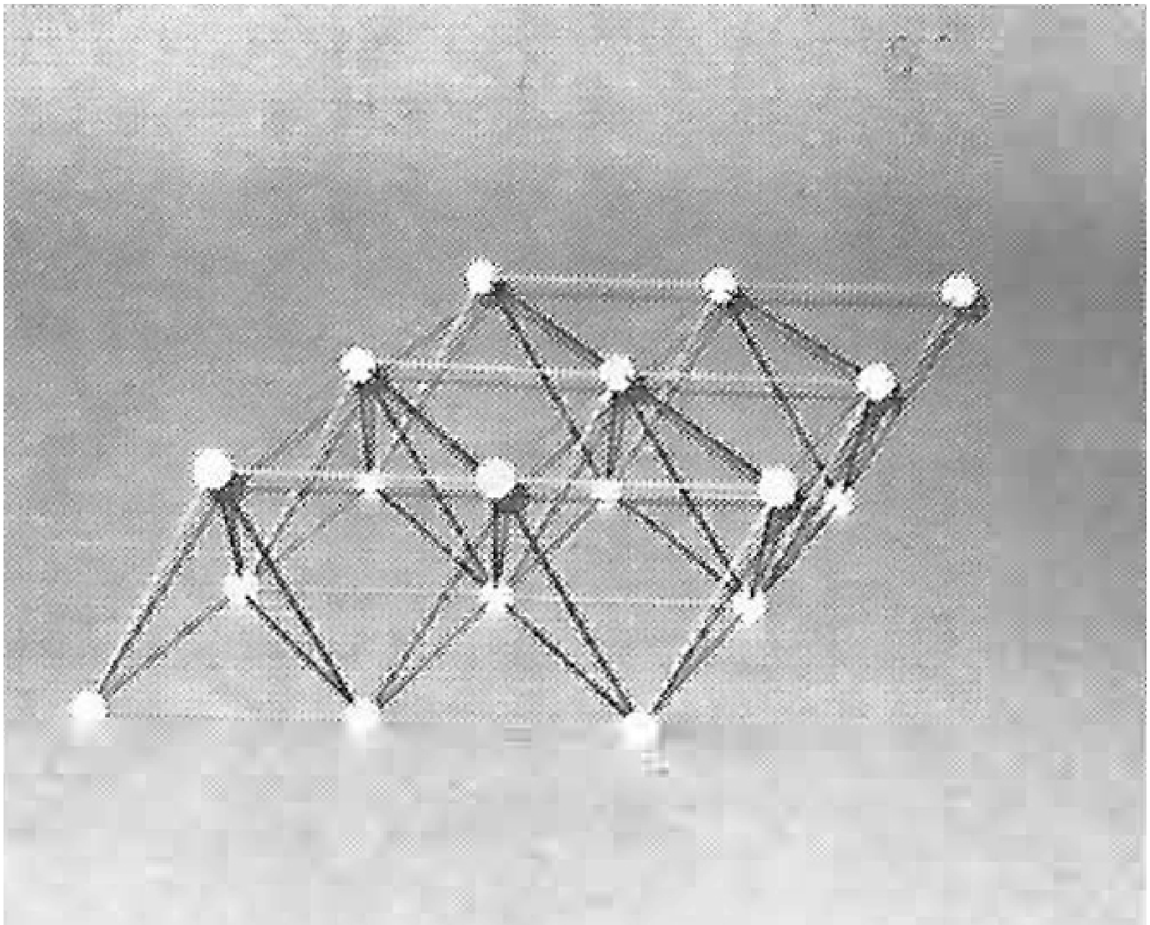


Figure 62: 4 A cells

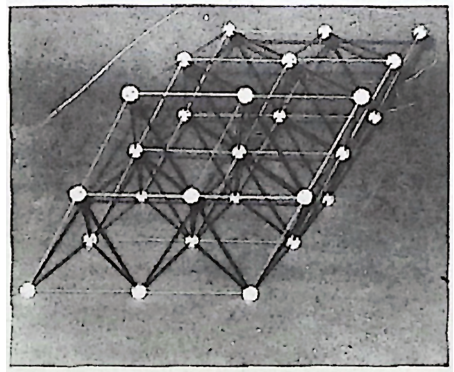


Figure 63: 8 A cells

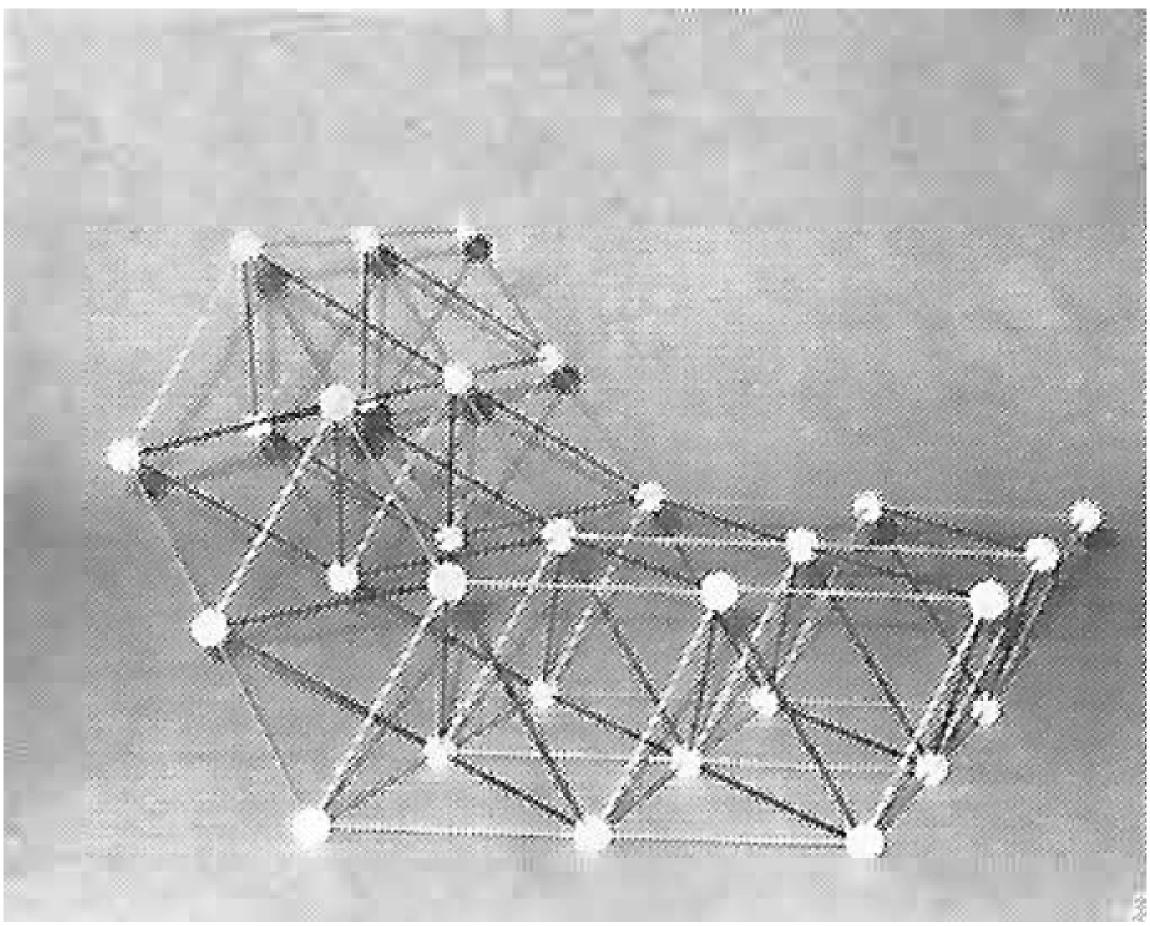


Figure 64: Truss turning corner 8 A cells

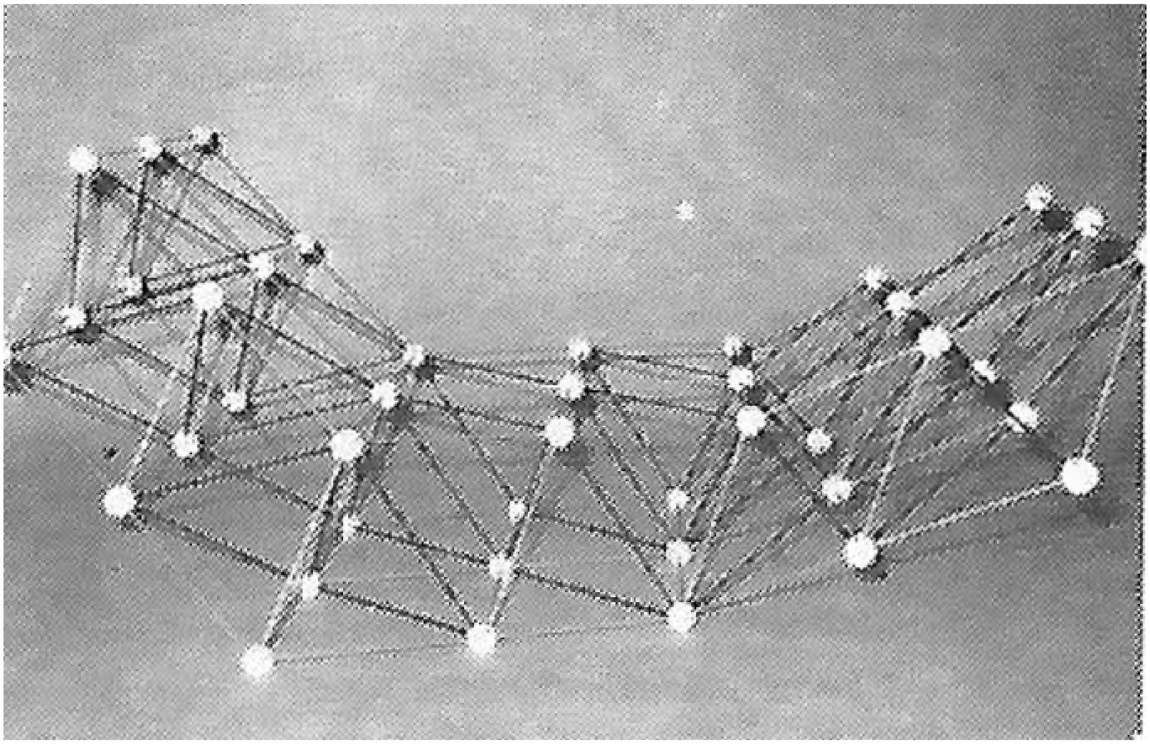


Figure 65: Truss turning corner 8 A cells - 4 B cells

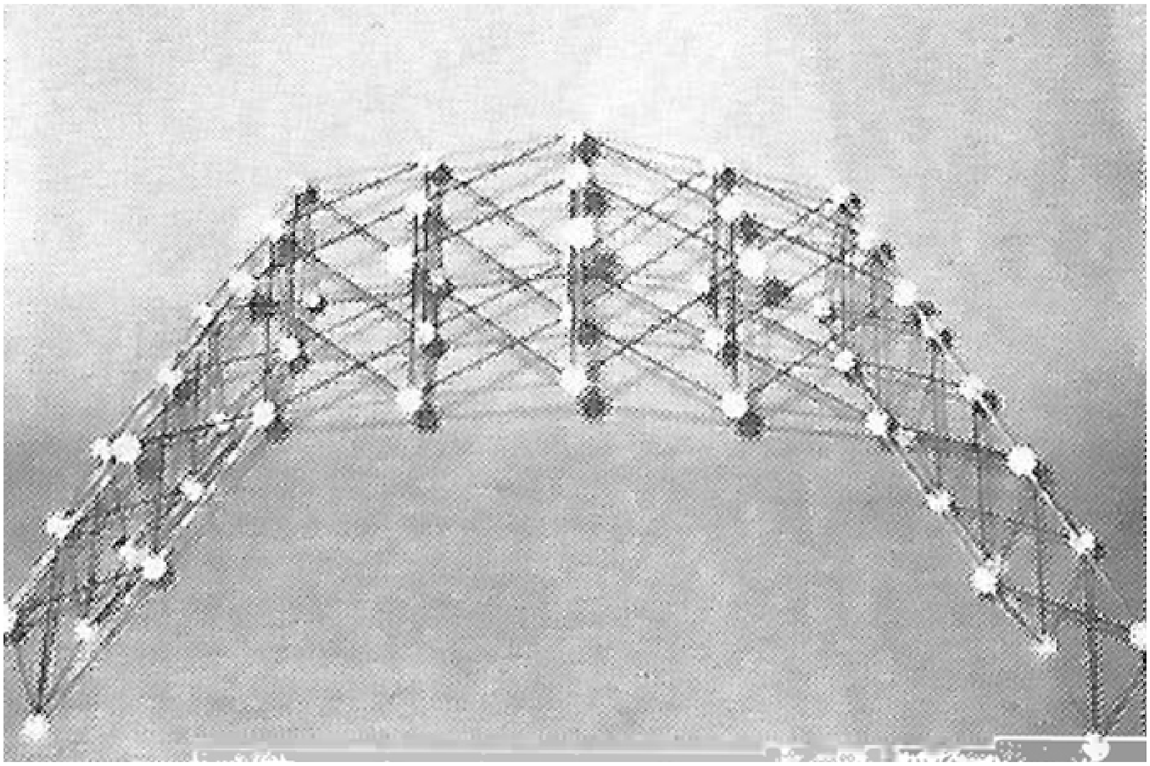


Figure 66: Truss forming arch* A cells middle sections B cells on sides

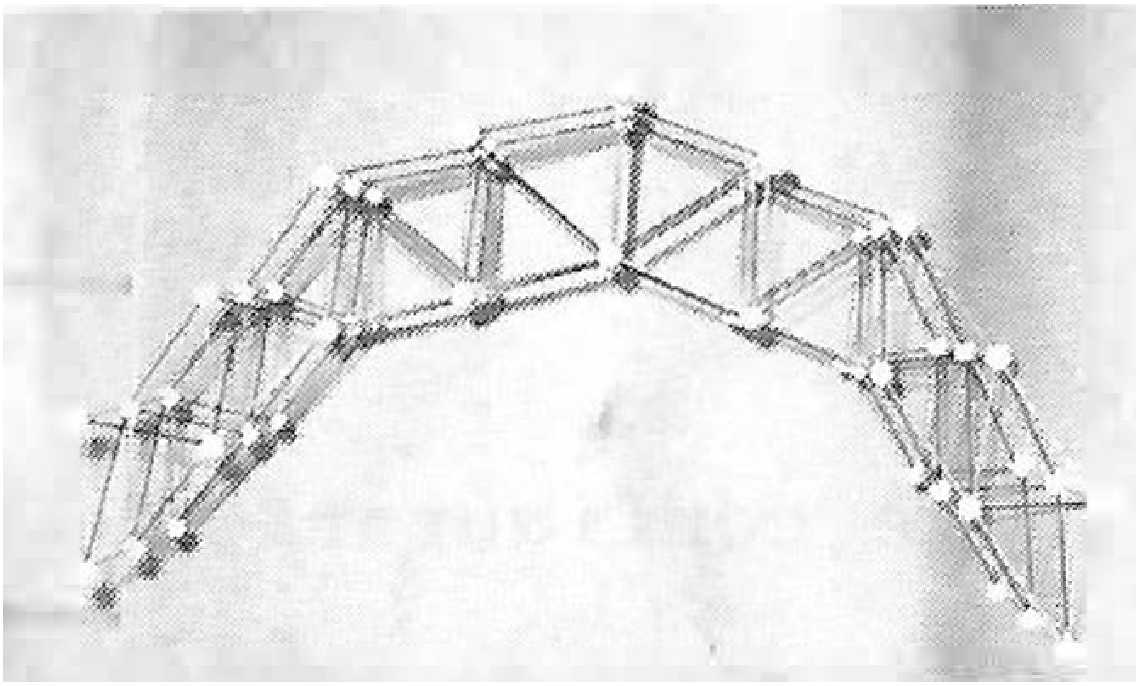


Figure 67: *Truss forming arch Head on view

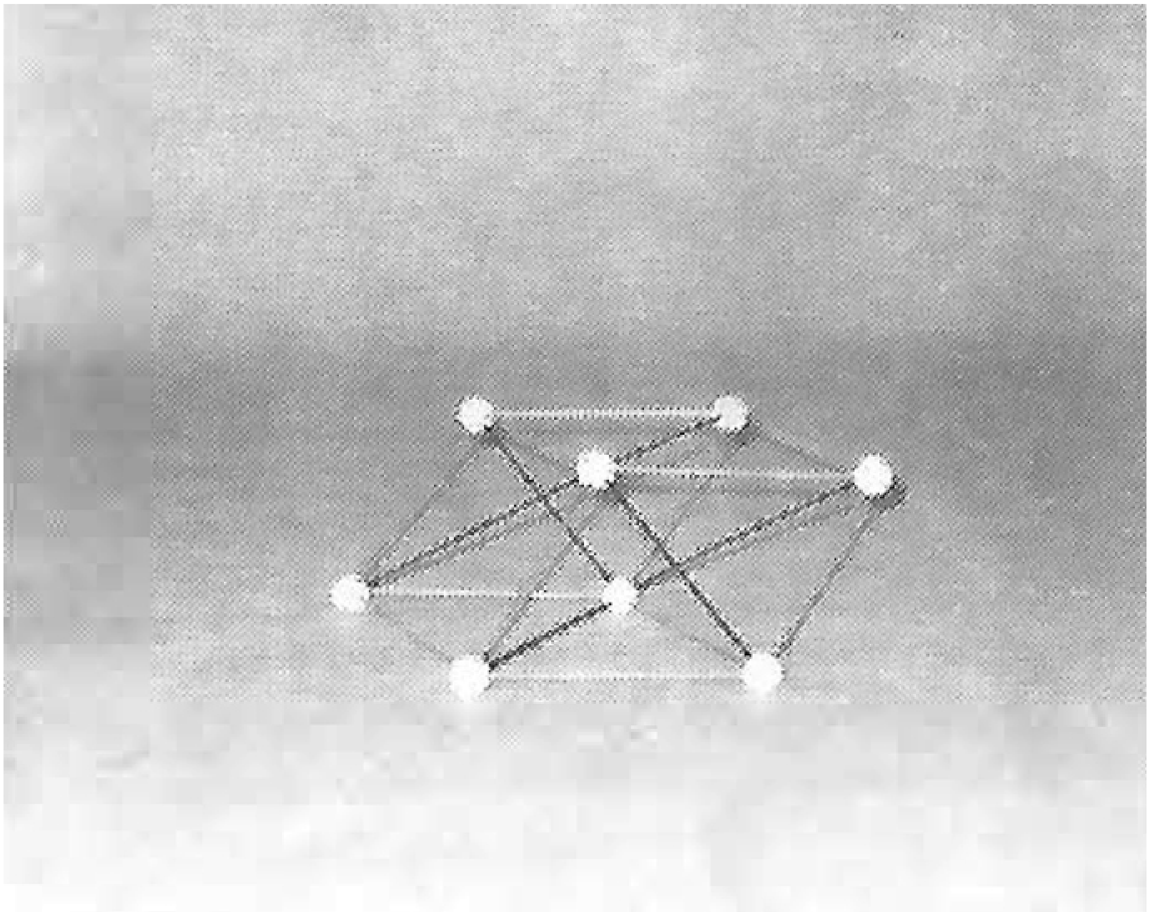


Figure 68: Single 21 zone B cell

21-zone truss forming a cap over 5 six-zone diamonds

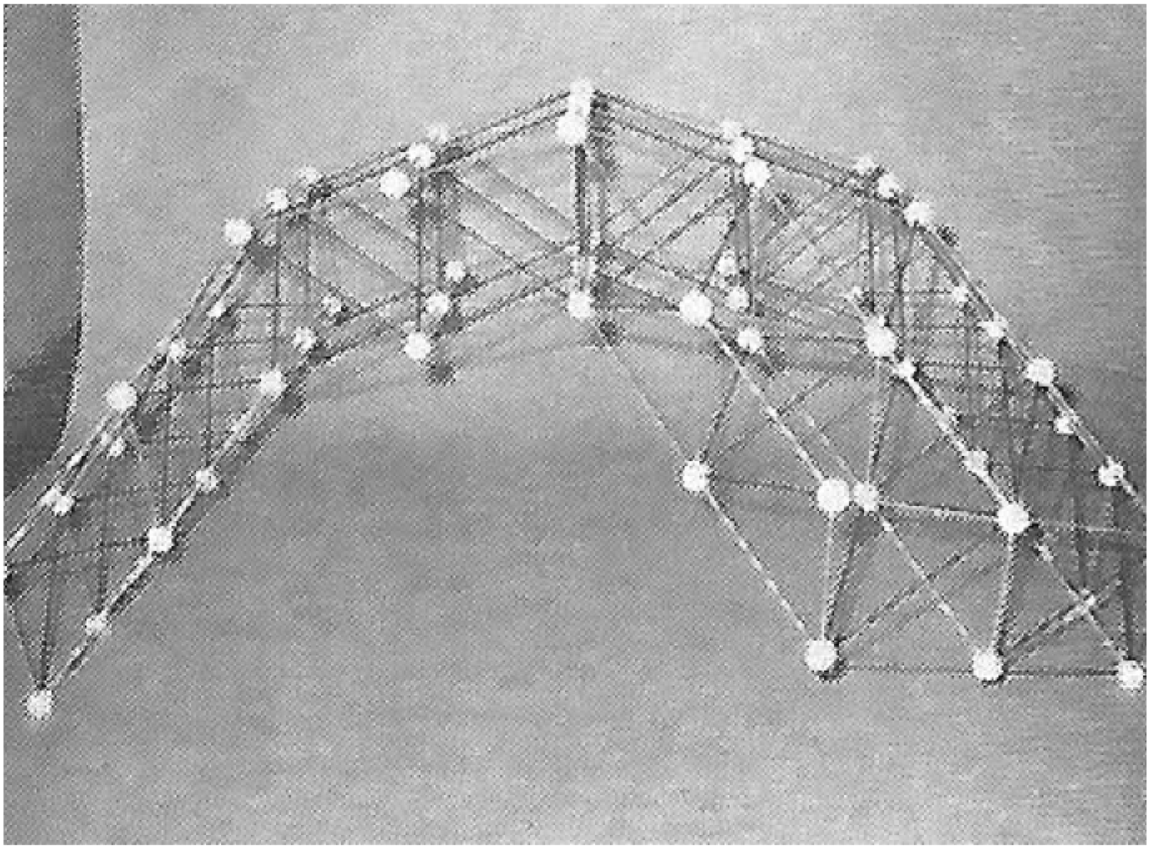


Figure 69

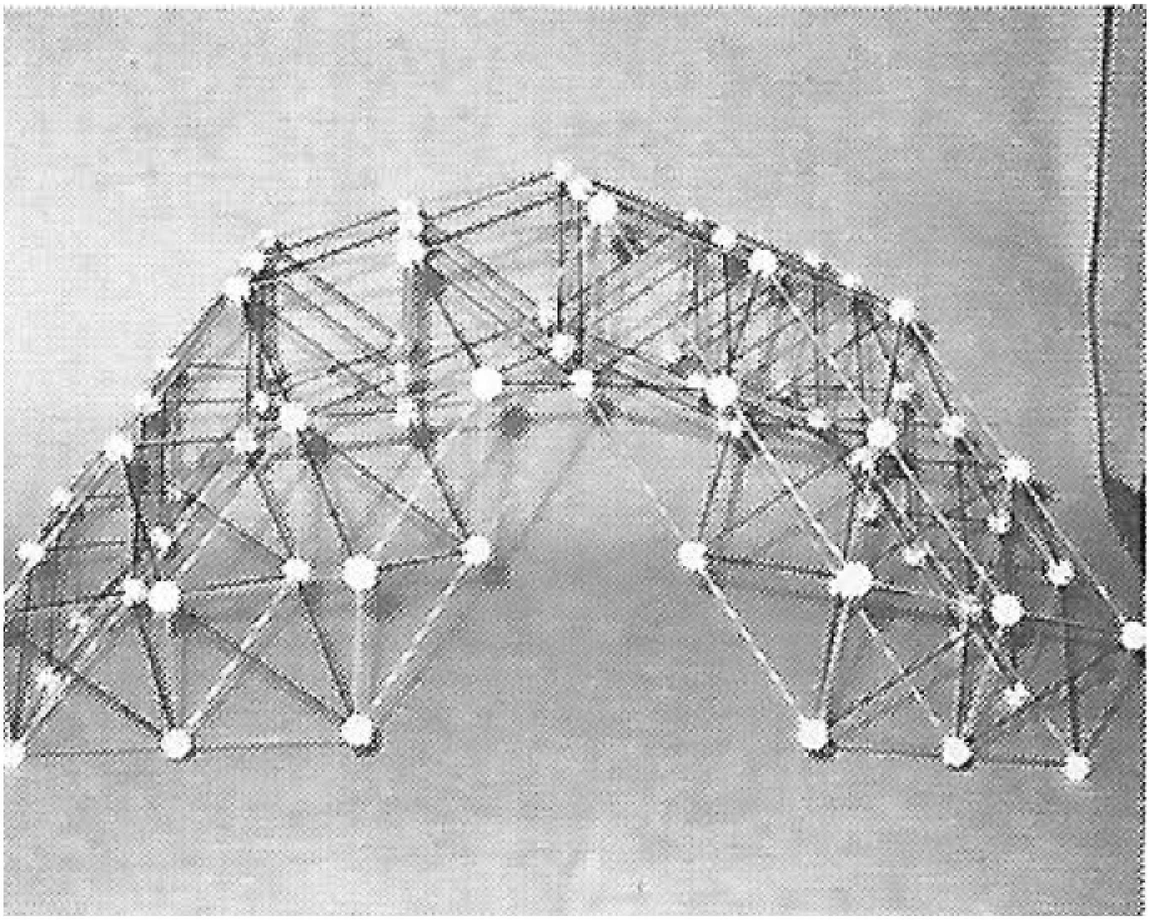


Figure 70

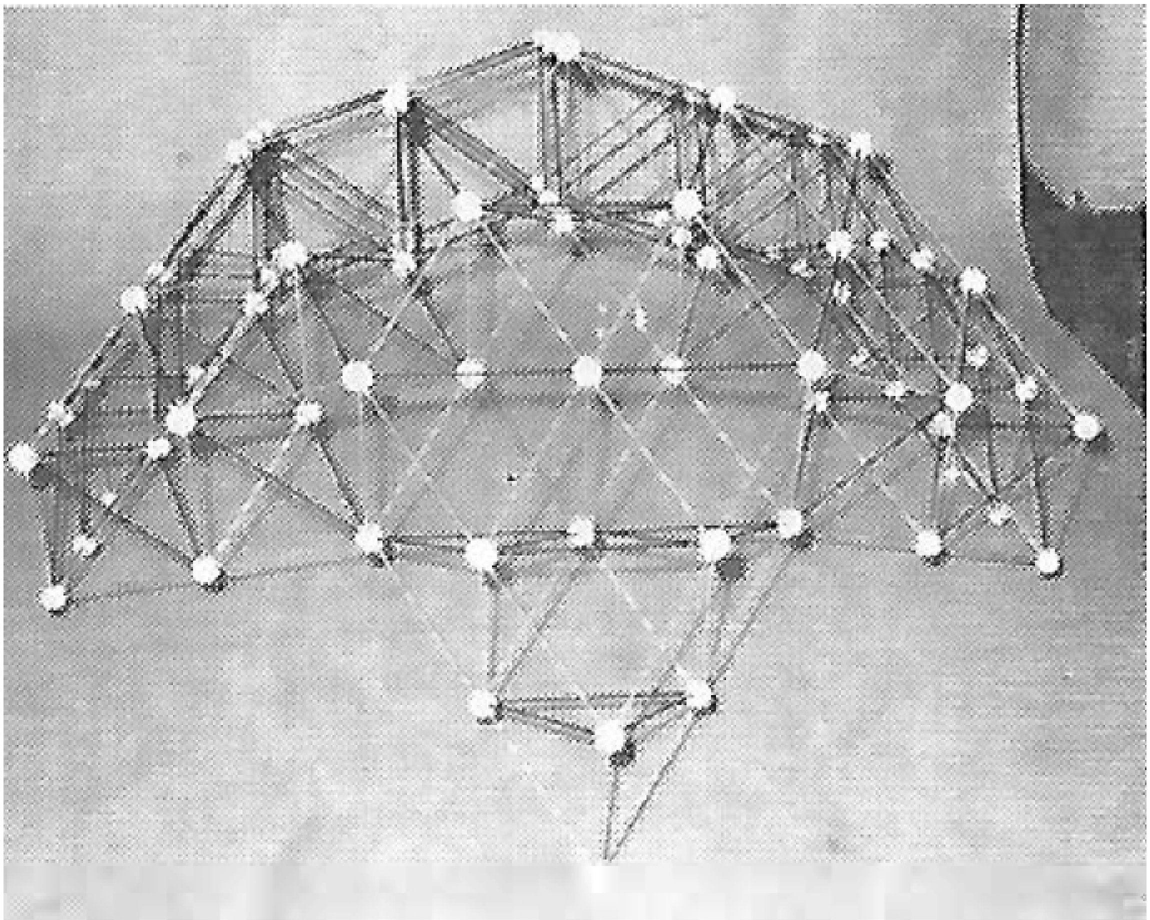


Figure 71

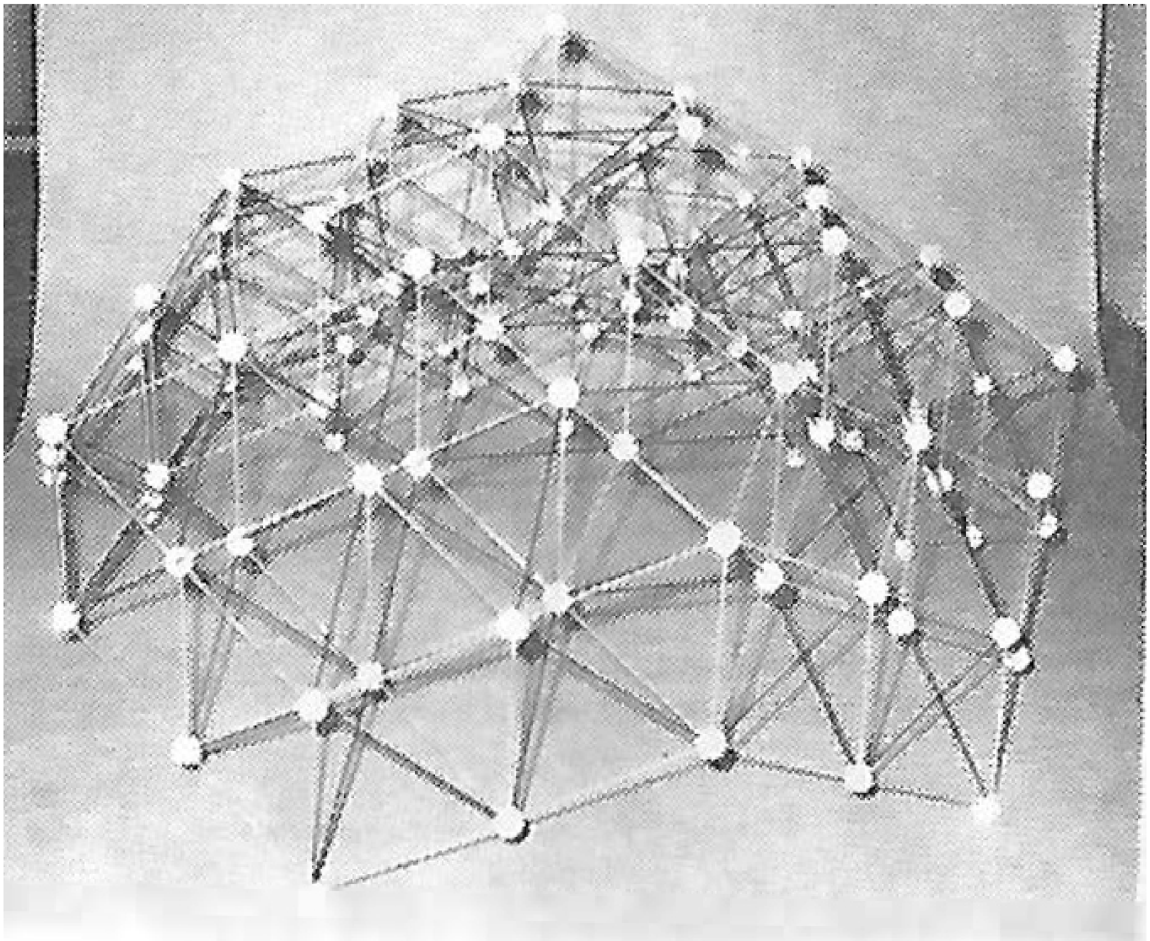


Figure 72: Six-zone truss covering the top half of triacontahedron

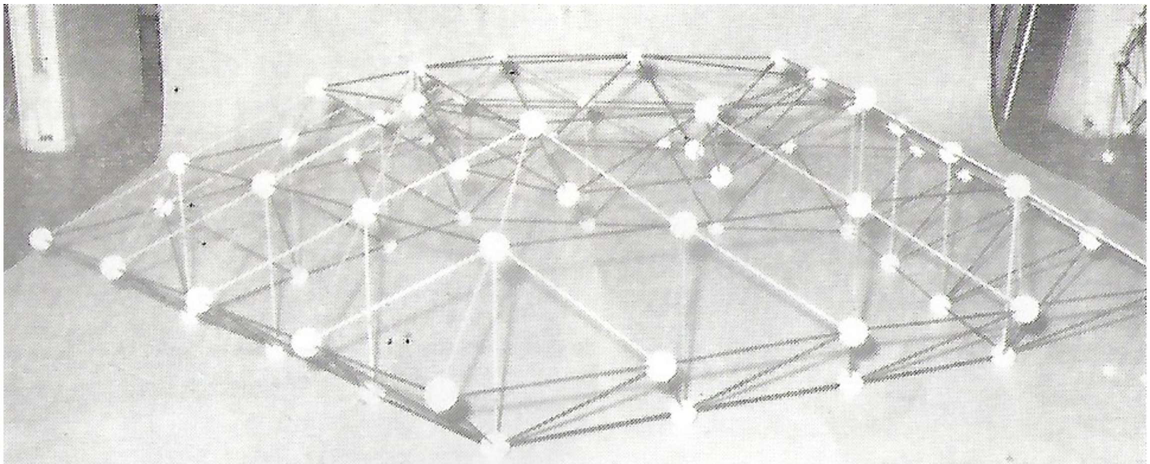


Figure 73: Pentagonal structure formed by truncating pent corner of triacontahedron. See structure in Figure 162

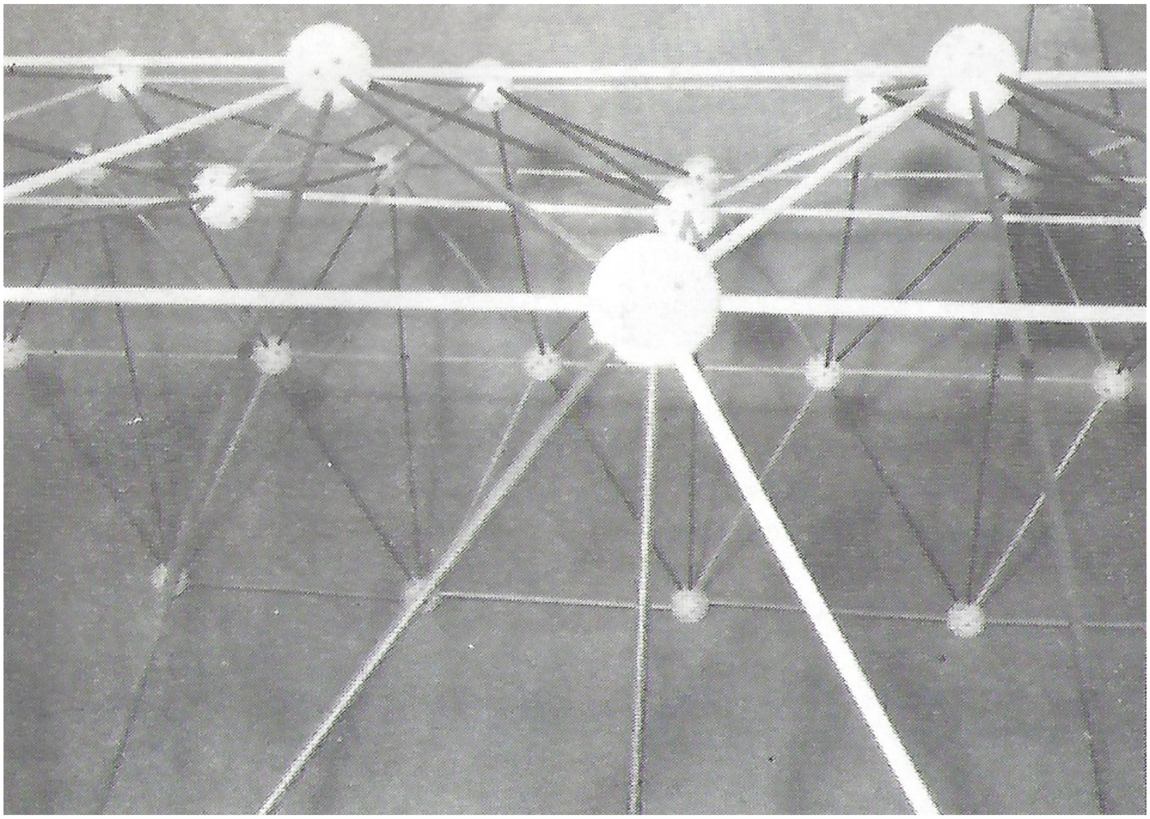


Figure 74

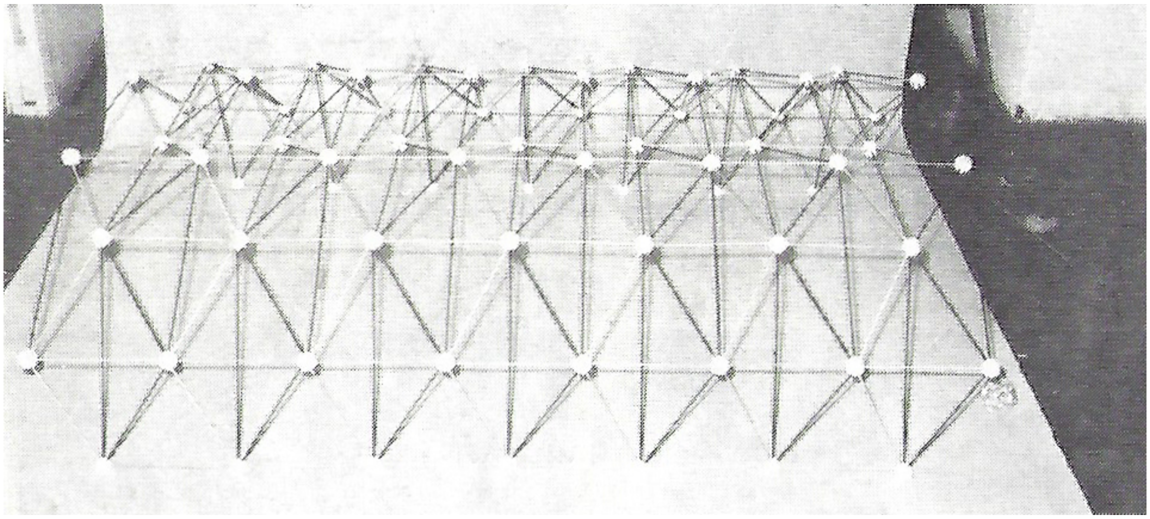


Figure 75: Rectangular truss - folded plate configuration

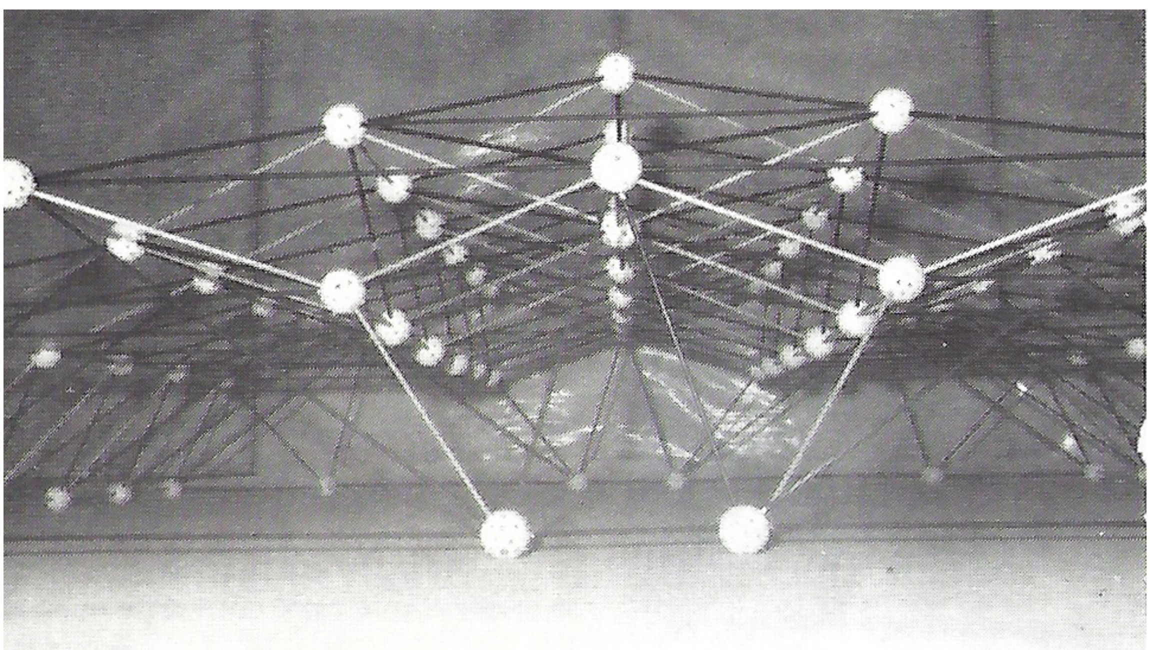


Figure 76: Truss formed of obtuse cells.

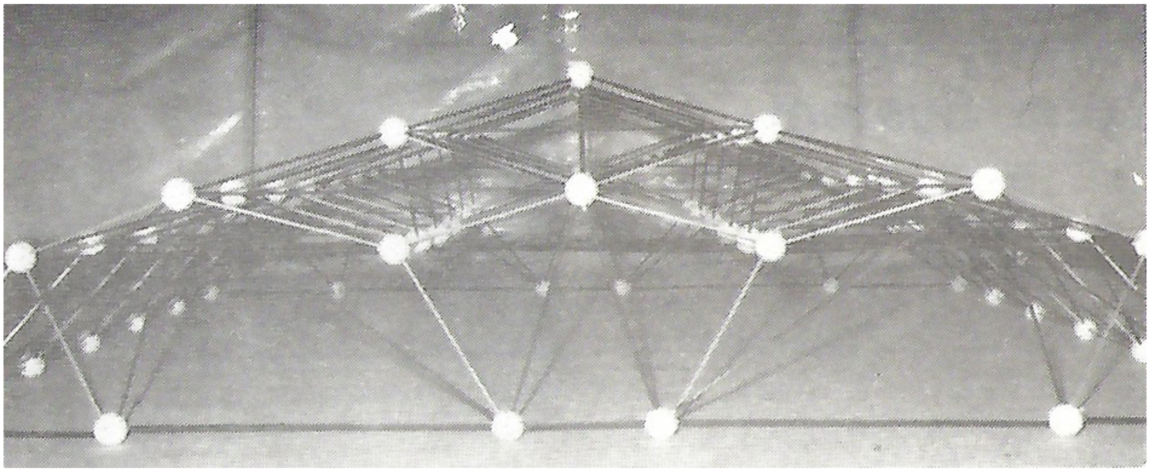


Figure 77

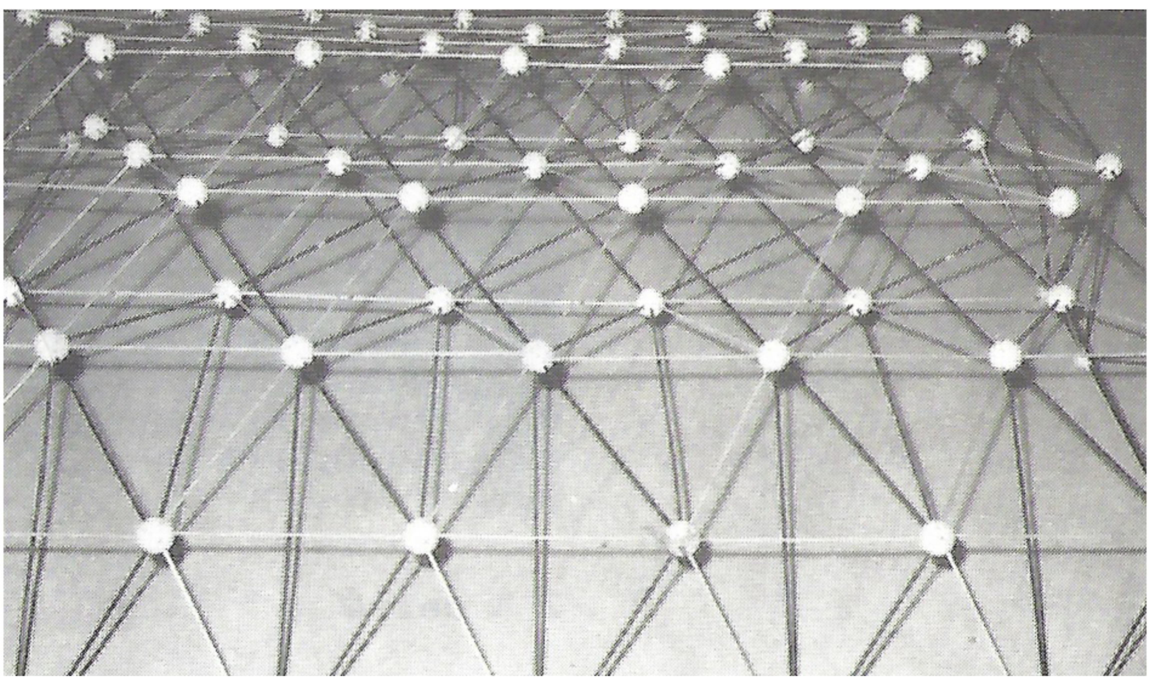


Figure 78: Two views of a rectangular trussed roof.

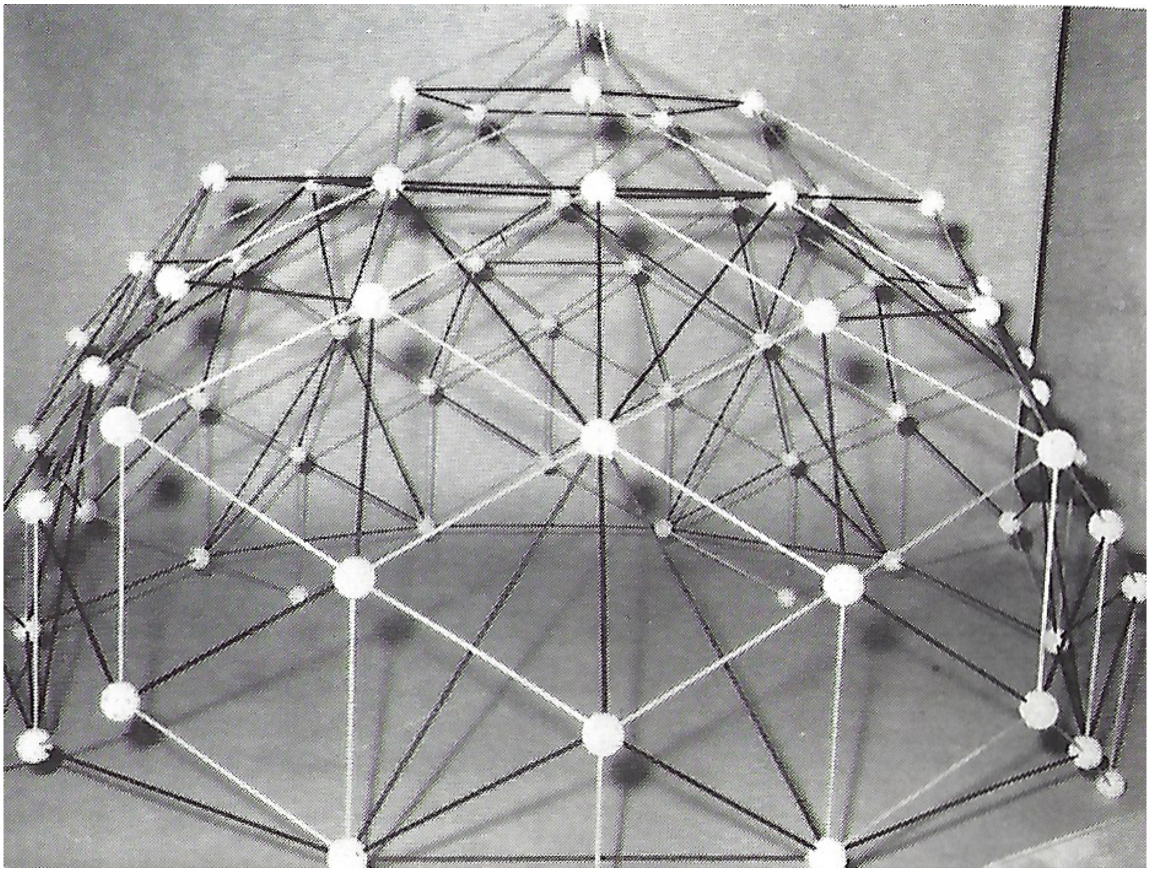


Figure 79: Triacontahedron—with interior icosadodecahedron for stiffening.

6.2 Triacontahedron

Radius to acute angled corner $= AT$.
Radius to obtuse angled corner $= BT$.

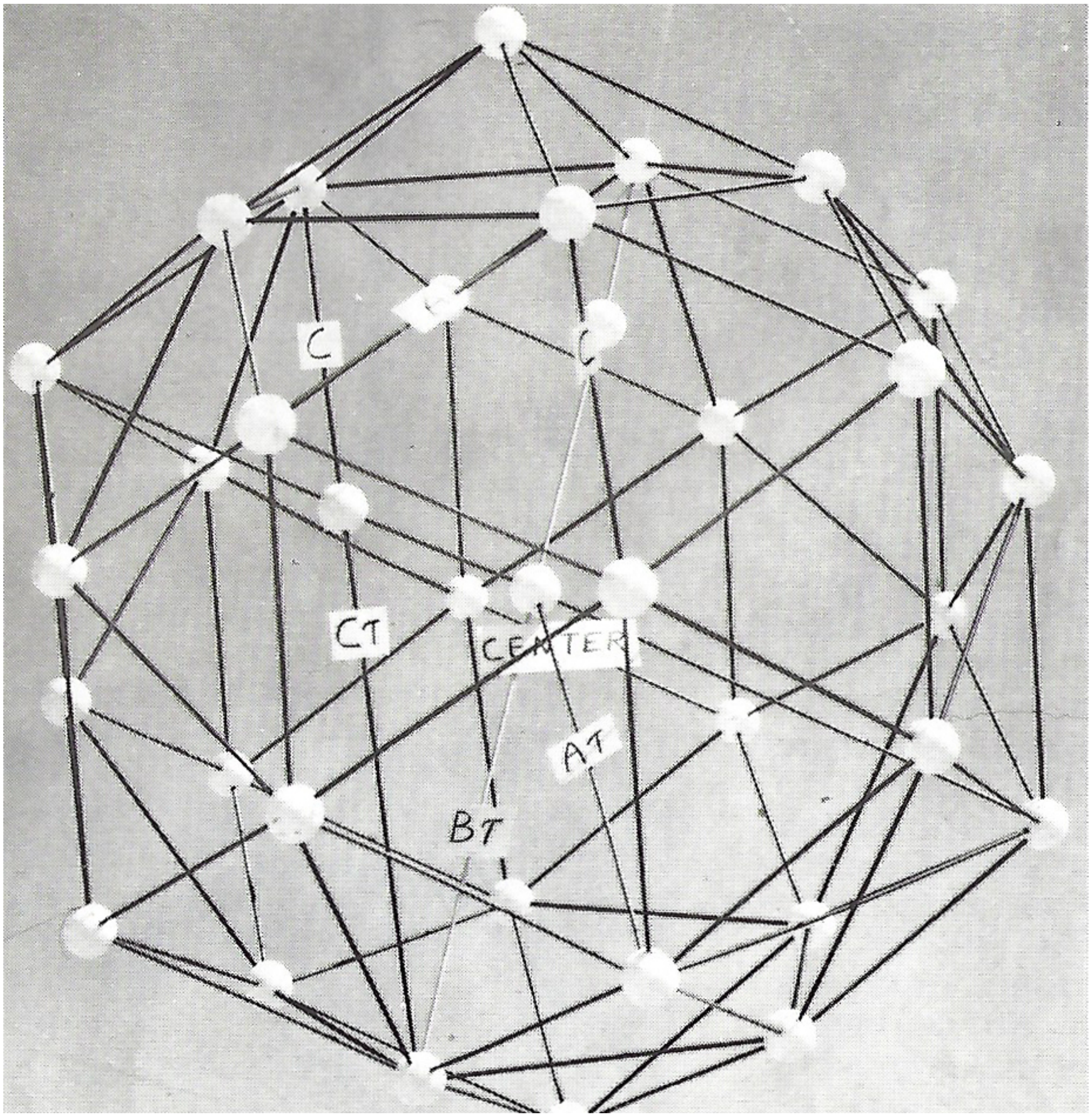


Figure 80: Triacontahedron

An enneacontahedron within a triacontahedron. The six sided vertices of the enneacontahedron coincide with the three sided vertices of the triacontahedron.

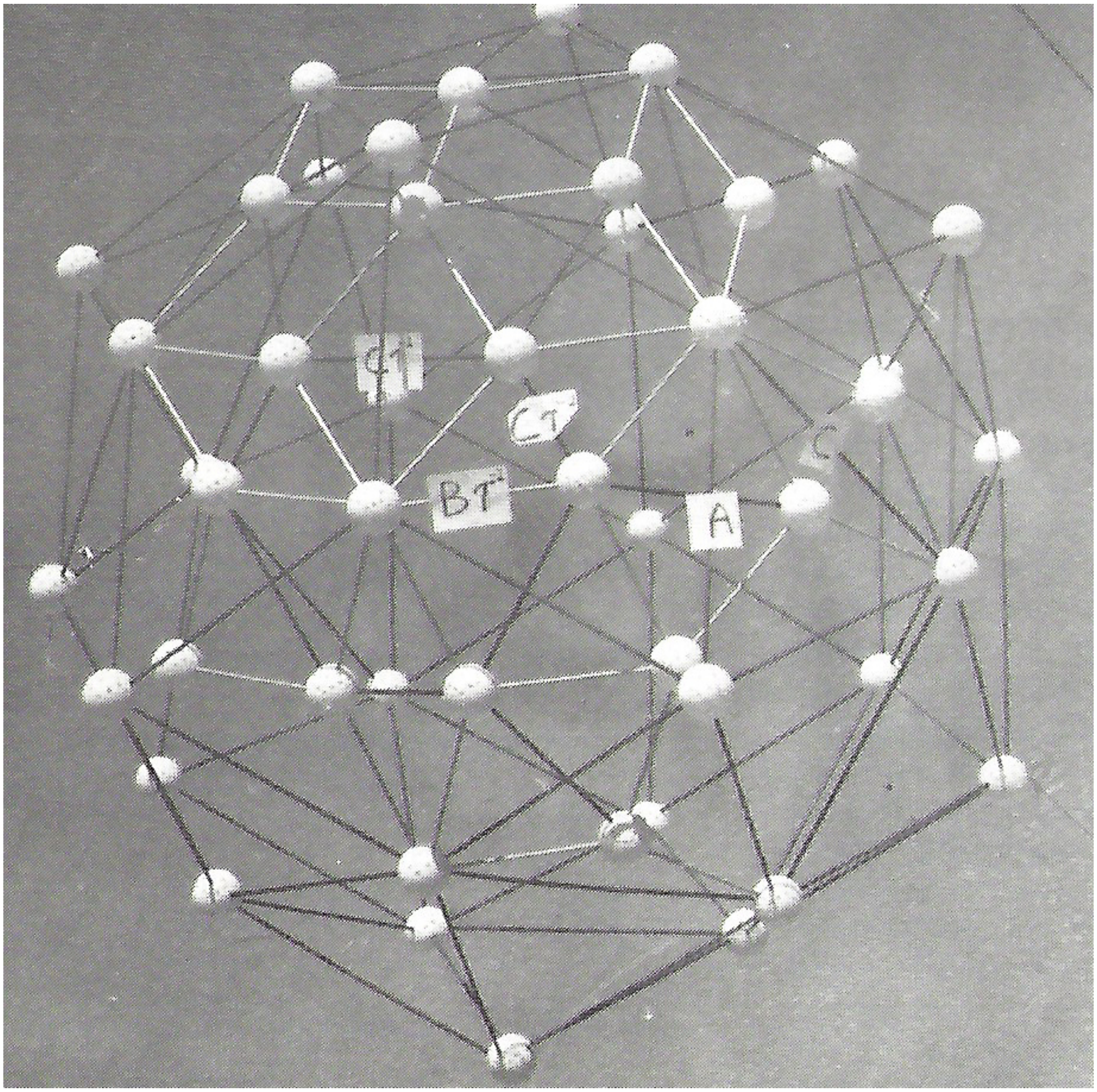


Figure 81: Enneacontahedron within a triacontahedron

Triacontahedron with edge $= A$ fused with Triacontahedron with edge $= AT^{-1}$.

Short diagonals of large diamond faces appear as long diagonals of small diamond faces.

The acute angled vertex of a large triacontahedron is located at the center of a small triacontahedron.

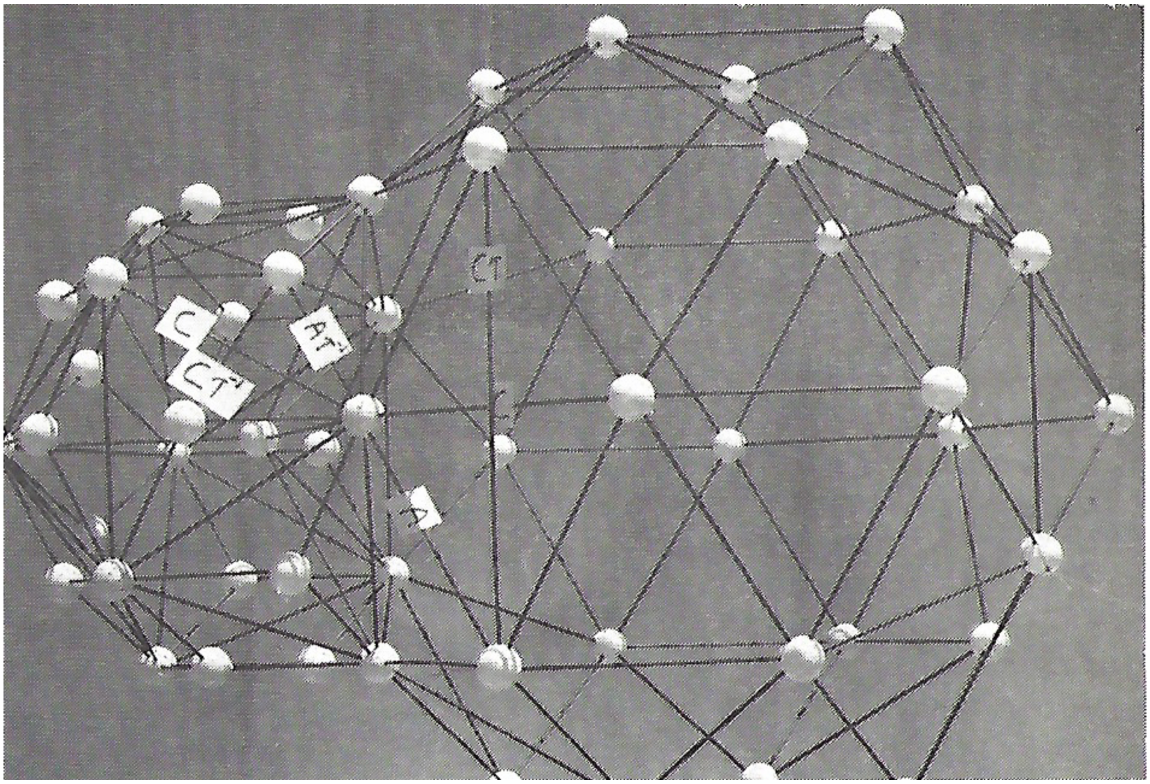


Figure 82: Enneacontahedron within a triacontahedron

Triacontahedron clustering carried from A to AT^{-2}.

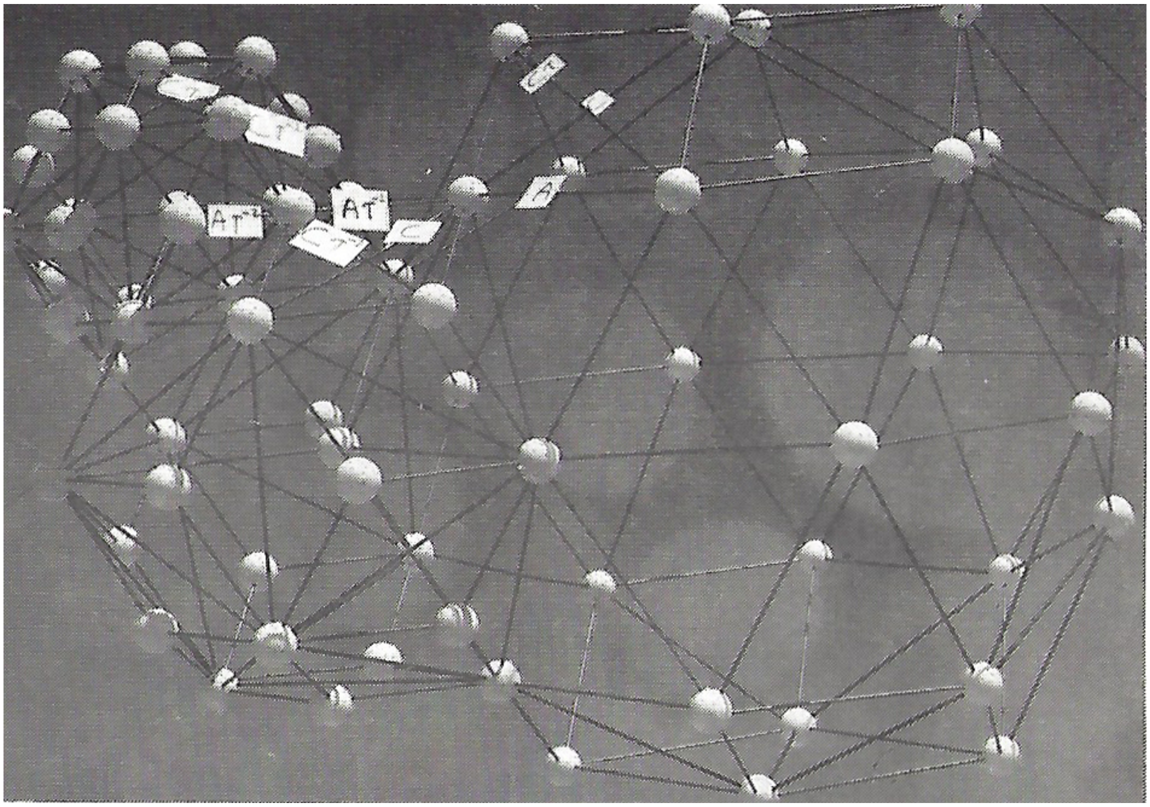


Figure 83: Triacontahedron

7 Critical Constants

7.1 Six Zone

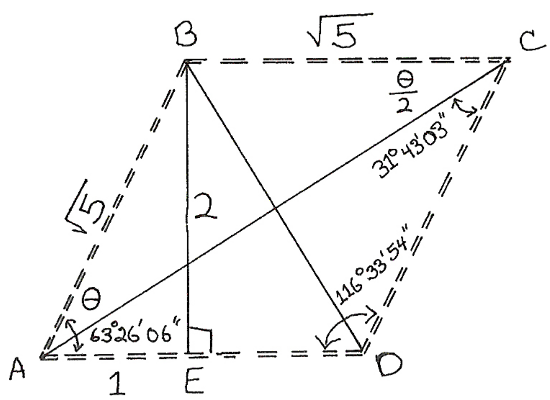


Figure 84: Six zone

$$\frac{\overline{BD}}{\overline{AB}} = 1.0514622$$

$$\frac{\overline{AC}}{\overline{AB}} = 1.7013016$$

$$\cos\theta = 0.4472136$$

$$\sin\theta = 0.8944272$$

$$\tan\theta = 2.0000000$$

$$\sin\frac{\theta}{2} = 0.5257310$$

$$\cos\frac{\theta}{2} = 0.8506507$$

$$\tan\frac{\theta}{2} = 0.6180339$$

7.2 Ten Zone

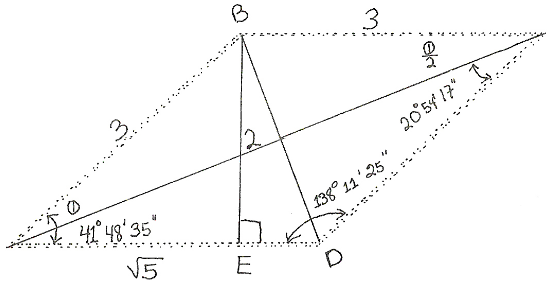


Figure 85: Ten zone

$$\frac{\overline{BD}}{\overline{AB}} = 0.7136441$$

$$\frac{\overline{AC}}{\overline{AB}} = 1.8683446$$

$$\cos\phi = 0.7453559$$

$$\sin\phi = 0.6666666$$

$$\tan\phi = 0.8944272$$

$$\sin\frac{\phi}{2} = 0.3568220$$

$$\cos\frac{\phi}{2} = 0.9341723$$

$$\tan\frac{\phi}{2} = 0.3819660$$

7.3 Ten Zone

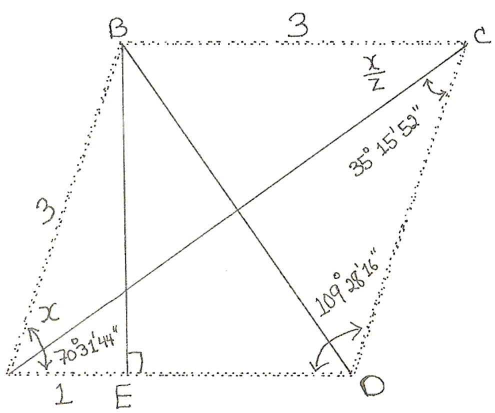


Figure 86

$$\frac{\overline{BD}}{\overline{AB}} = 1.1547005$$

$$\frac{\overline{AC}}{\overline{AB}} = 1.6329931$$

$$\cos X = 0.3333333$$

$$\sin X = 0.9428090$$

$$\tan X = 2.8284271$$

$$\sin\frac{X}{2} = 0.8164965$$

$$\cos\frac{X}{2} = 0.5773502$$

$$\tan\frac{X}{2} = 0.7071067$$

A line radii of an icosahedron with edge $= C$. The pair of radii outline one end of a golden diamond.

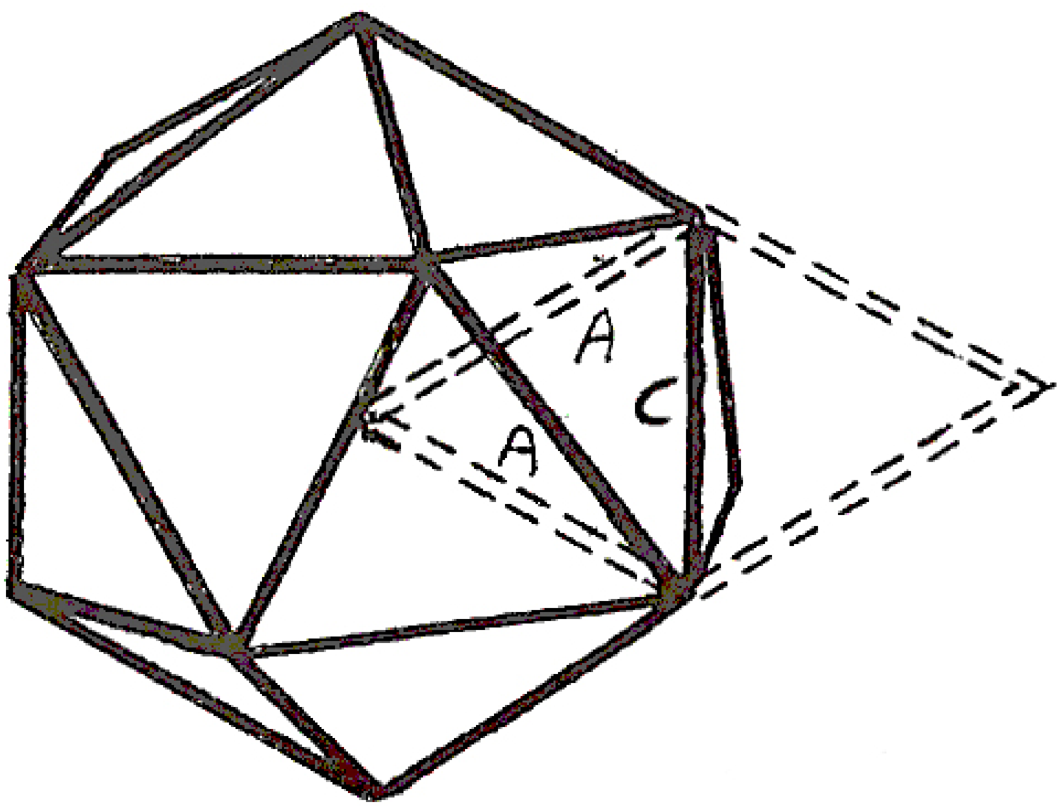


Figure 87: Six-Zone Diamond

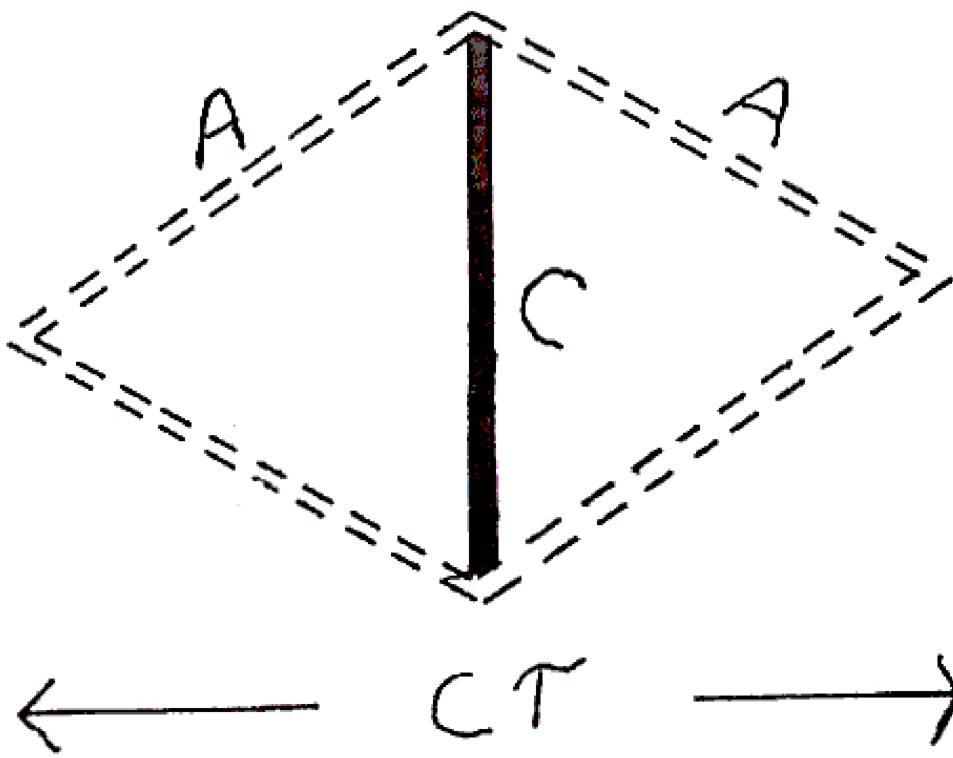


Figure 88: Golden Diamond

B line radii of dodecahedron with edge $= C$. One pair outlines one end of the skinny diamond; another pair outlines an end of a maraldi diamond.

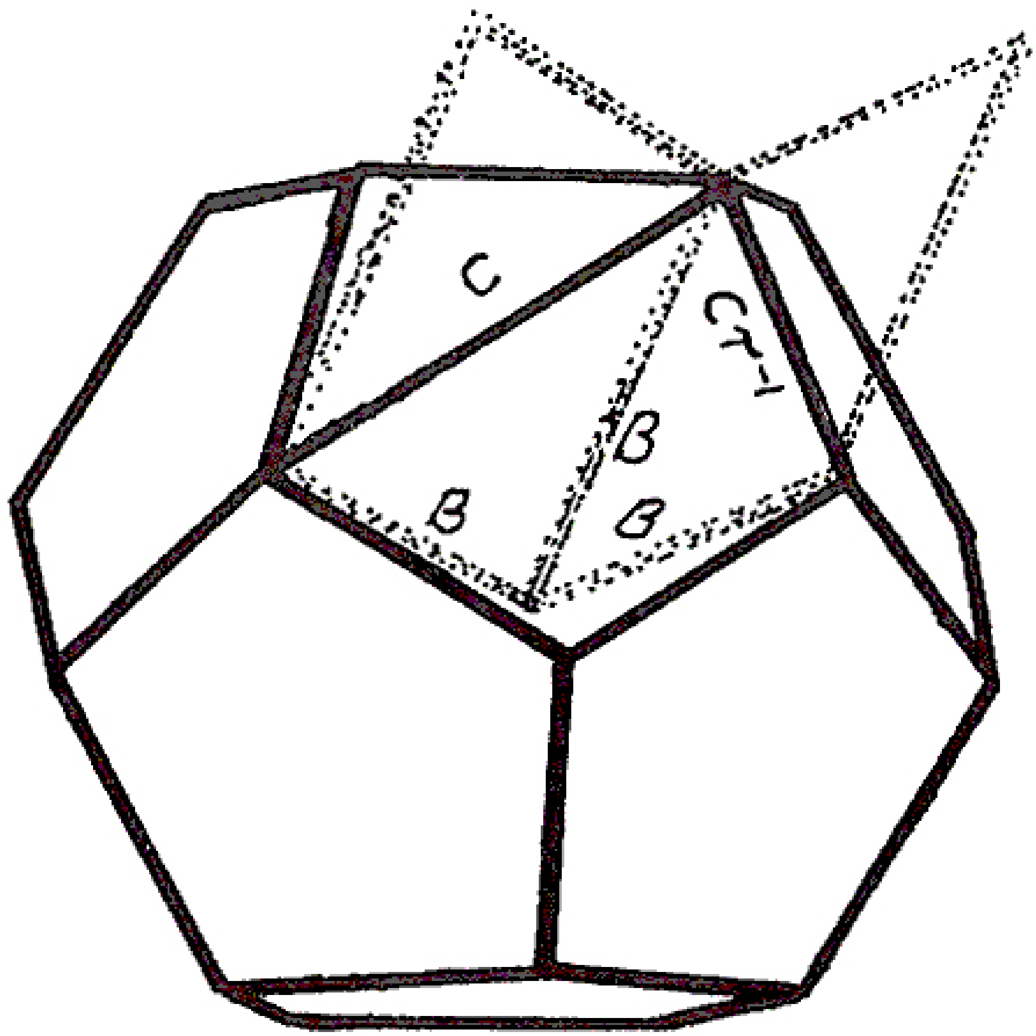


Figure 89: 10 Zone Diamonds

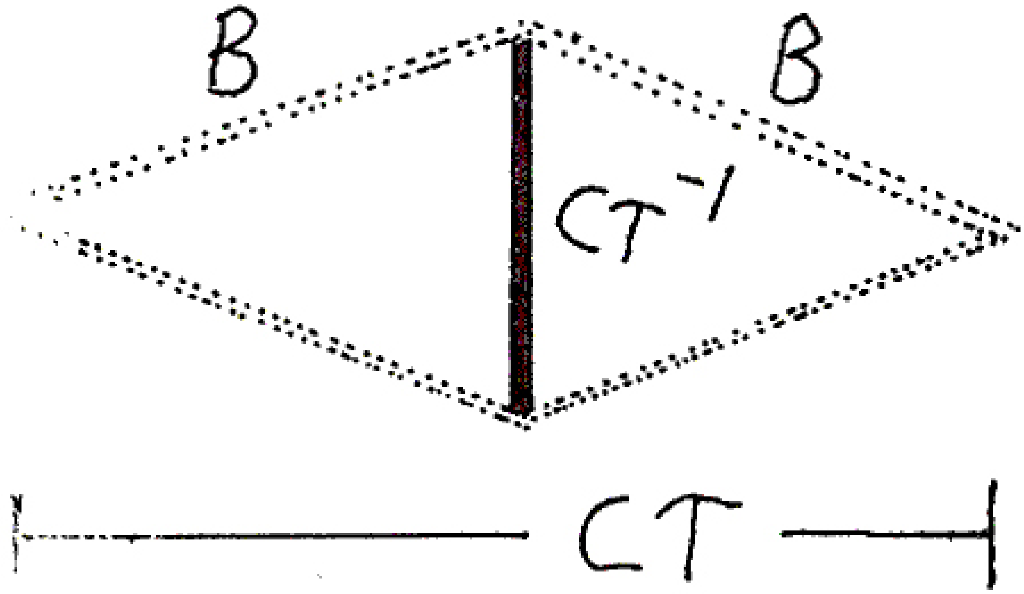


Figure 90: Skinny Diamond

$$A = 0.9510565\ C = \cos 18^\circ\ C$$
$$A = 1.0981855\ B$$

$$B = 0.8660254\ C = \cos 30^\circ\ C$$
$$B = 0.9105930\ A$$

$$C = 1.0514622\ A$$
$$C = 1.1547005\ B$$

Table 7.1: Proportions Of A, B, and C Lines

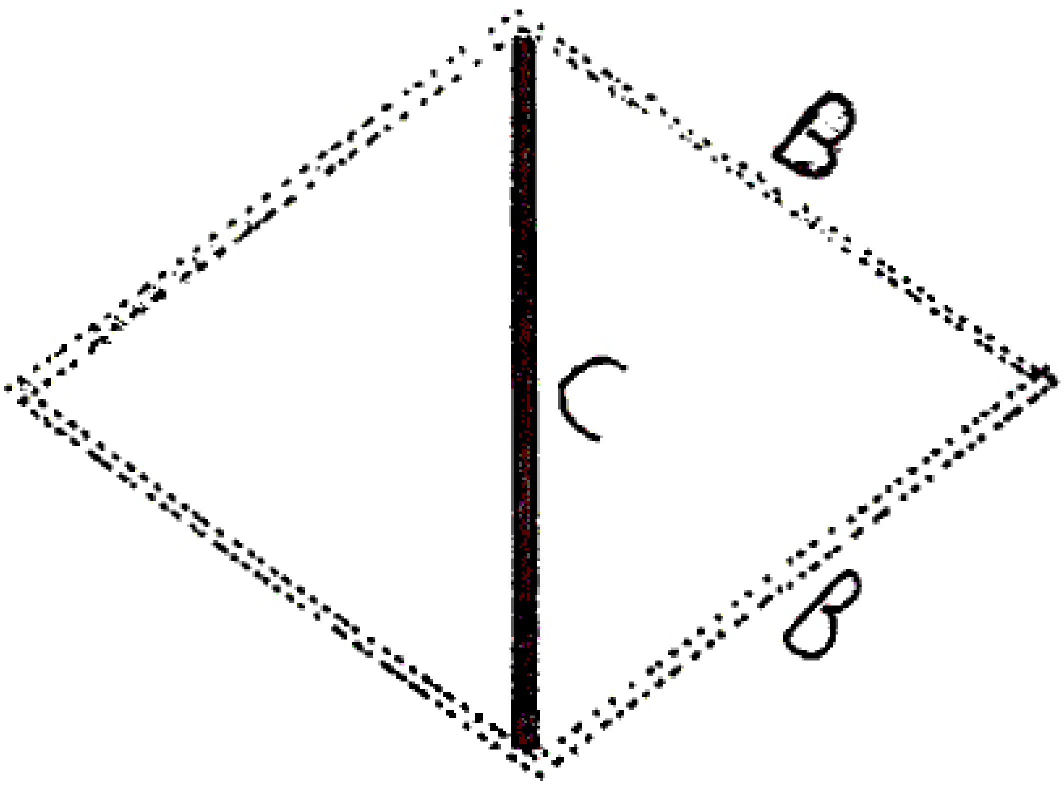


Figure 91: Maraldi Diamond

7.4 The Appearance of the CT^n Series in the Dimensions of the Triacontahedron and the Enneacontahedron

Both these patterns can be found as sub patterns of the five-fold symmetry patterns of page.

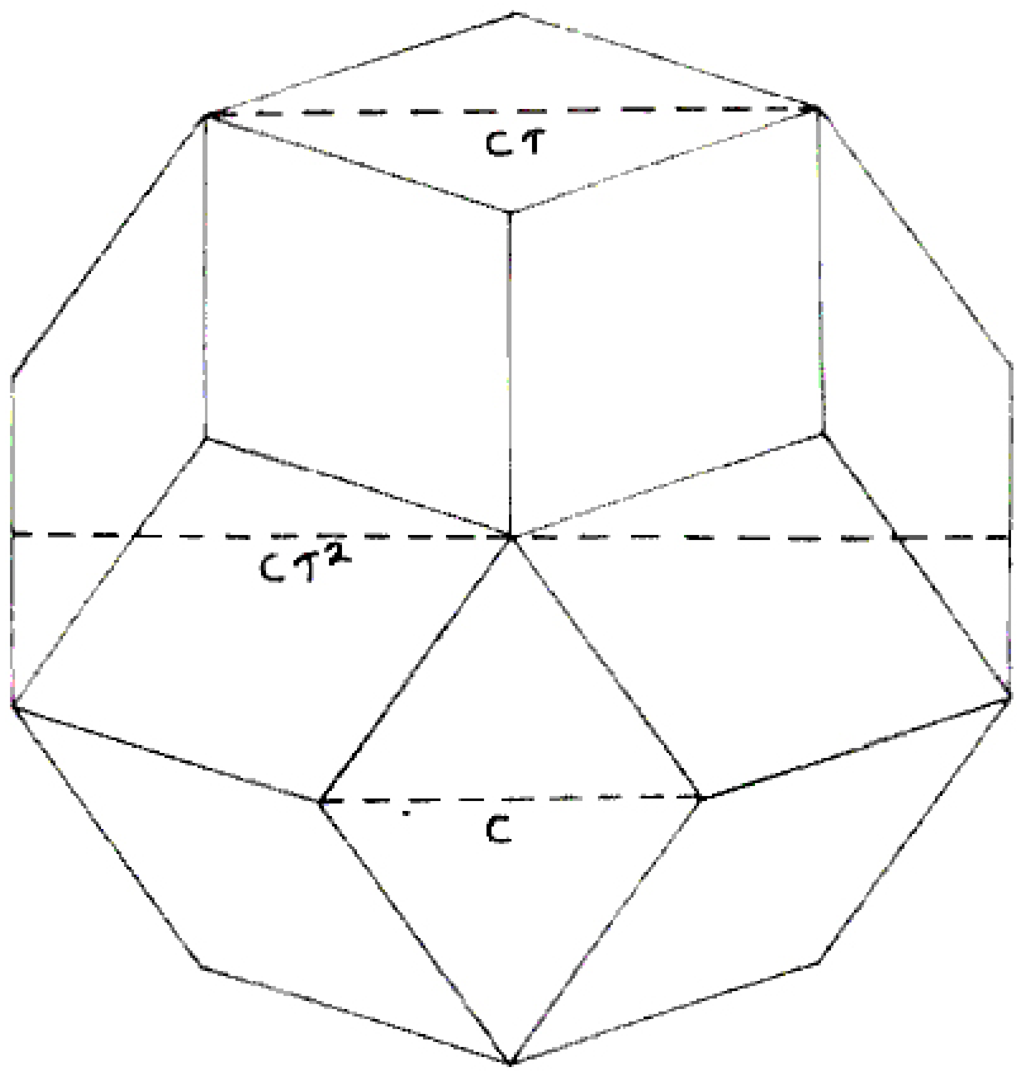


Figure 92: Triacontahedron

All edges are A lines—one A line is perpendicular to the plane of page.

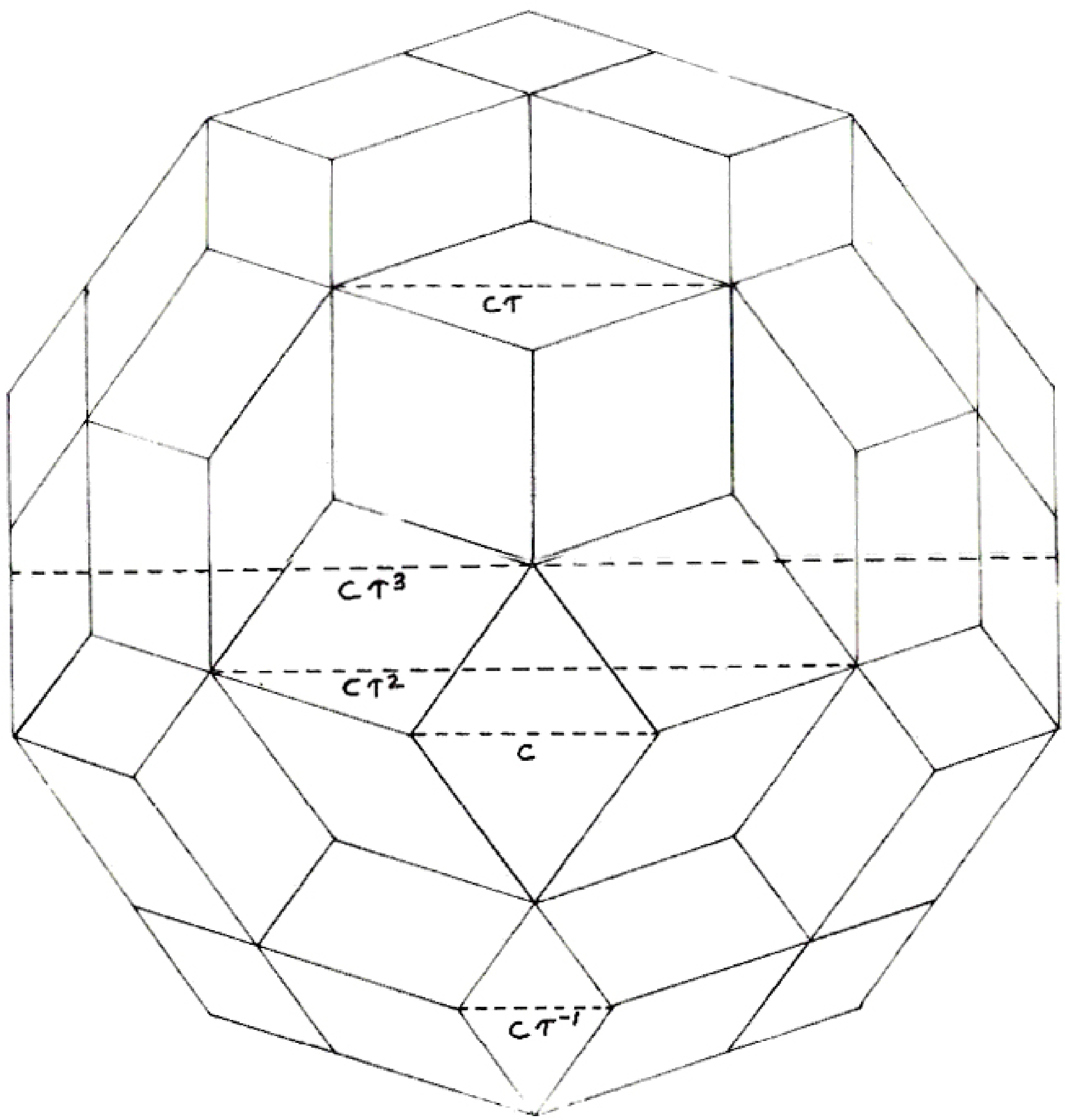


Figure 93: Enneacontahedron

All edges are B lines— Aline is perpendicular to the plane of page.

8 Symmetries of a Regular Thirty-one Zone Star

The dodecahedron and the icosahedron are duals of each other - the vertices of one match the face midpoints of the other and vice versa.

The ten B lines of the thirty-one zone star go through the vertices of the dodecahedron or, equivalently, the face midpoints of the icosahedron while the six A lines go through the vertices of the icosahedron or, equivalently, the face midpoints of the dodecahedron.

The fifteen C lines go through the edge midpoints of either the icosahedron or the dodecahedron.

The middles of edges are commonly midway between vertices or face midpoints and C lines bisect all angles between A lines and three of the four kinds of angles formed between B lines.

In examining angles between lines, we are also examining equators. There are six different equators and slicing through them, we form the R, S, T, V, X, and Y sections.

All pairs of lines lie in one of-these six kinds of sections.

See Sections for a discussion of different angles and the polygons they form.

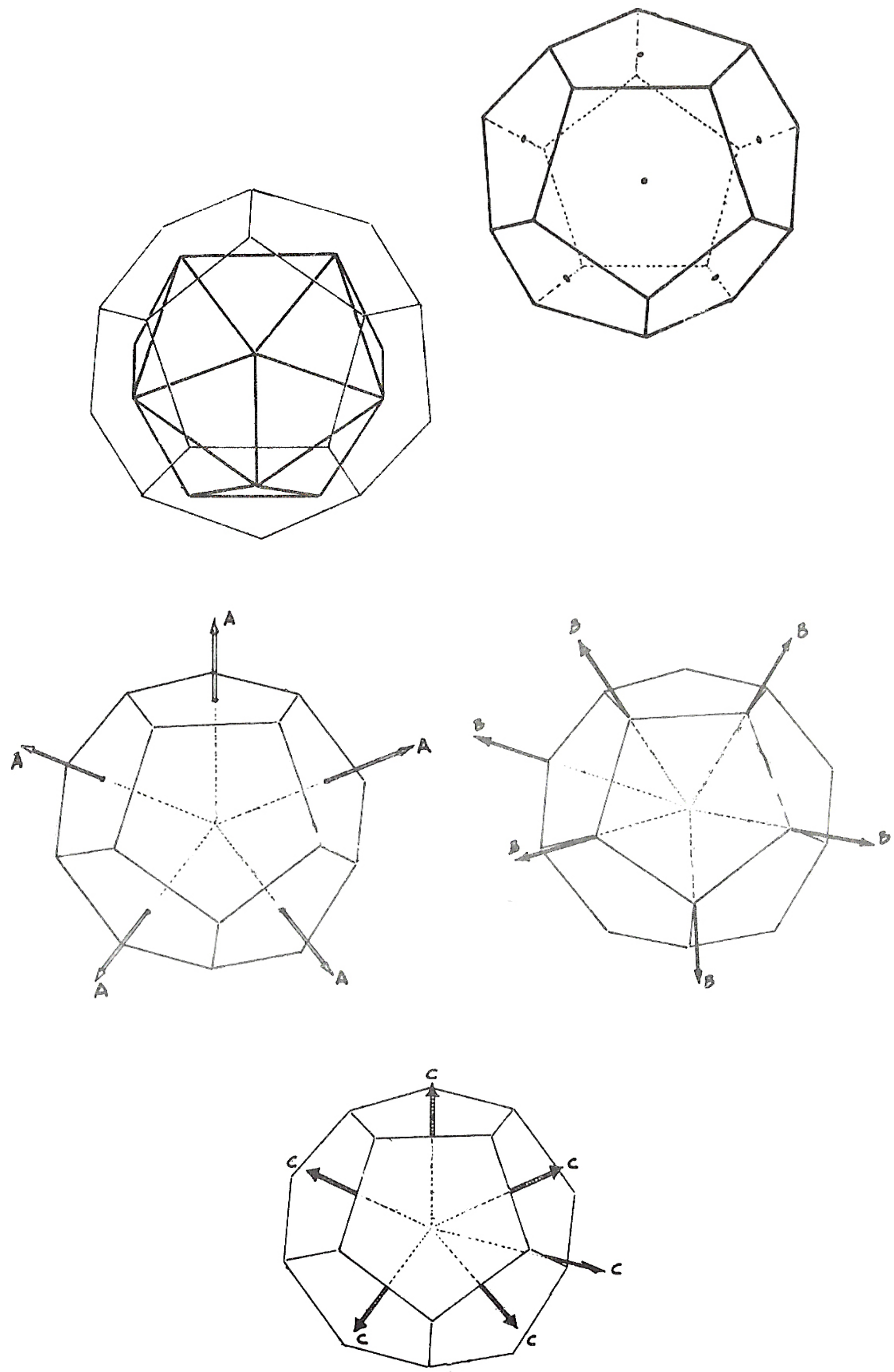


Figure 94: Thirty-One Zone Star: Dodecahedron

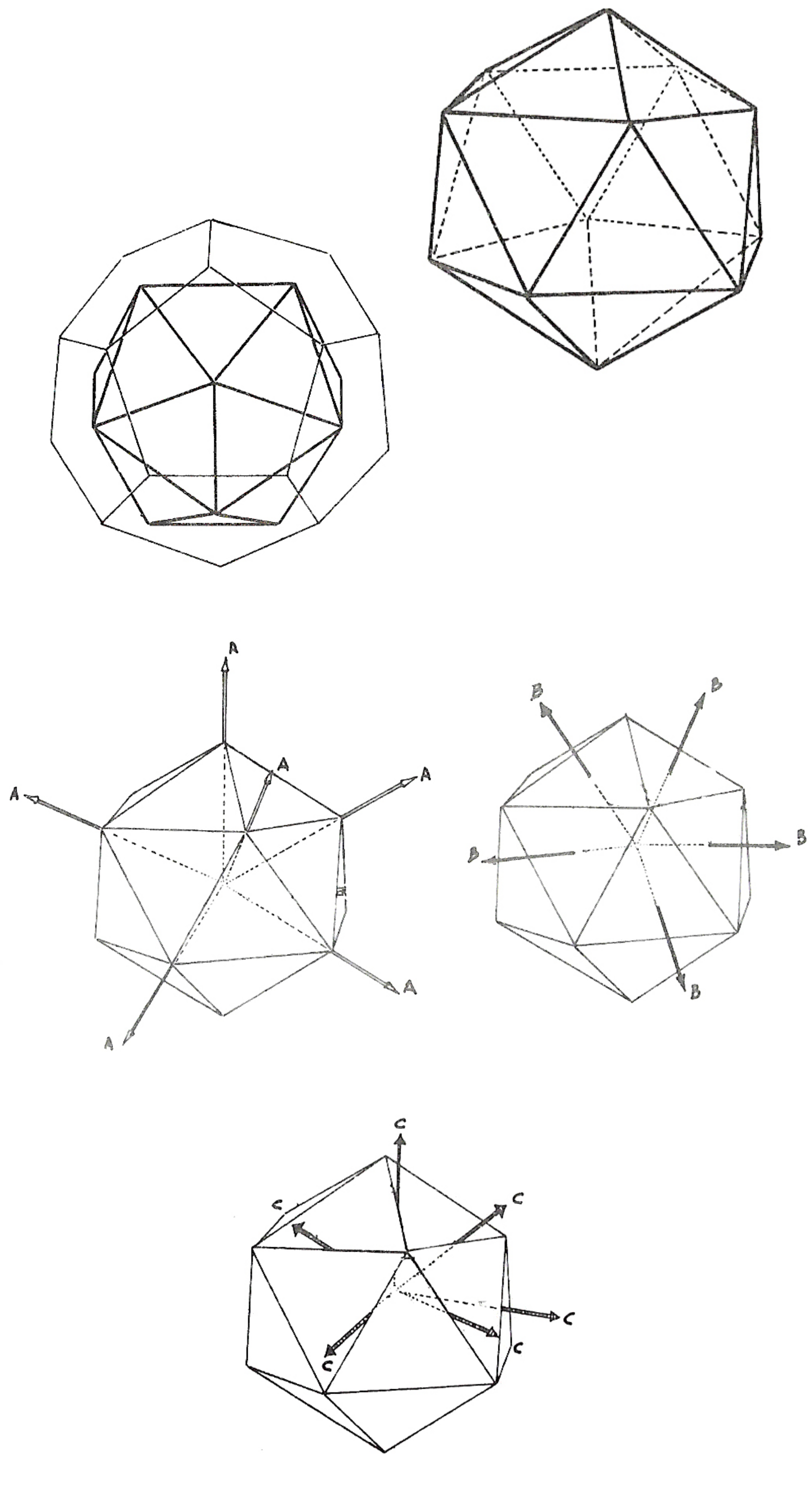


Figure 95: Thirty-One Zone Star: Icosahedron

9 Five-Fold Symmetry

The icosahedron and the dodecahedron have five-fold symmetry. They cannot occur as crystals. Crystals are built up of molecules that are located in systems of regular points. It is impossible for a system of regular points to have five-fold symmetry. The inability of objects with five-fold symmetry to fit together is obvious if one tries to it regular pentagonal tiles together to cover a plane. Three, four and six sided tiles will fit, but not regular five sided tiles.

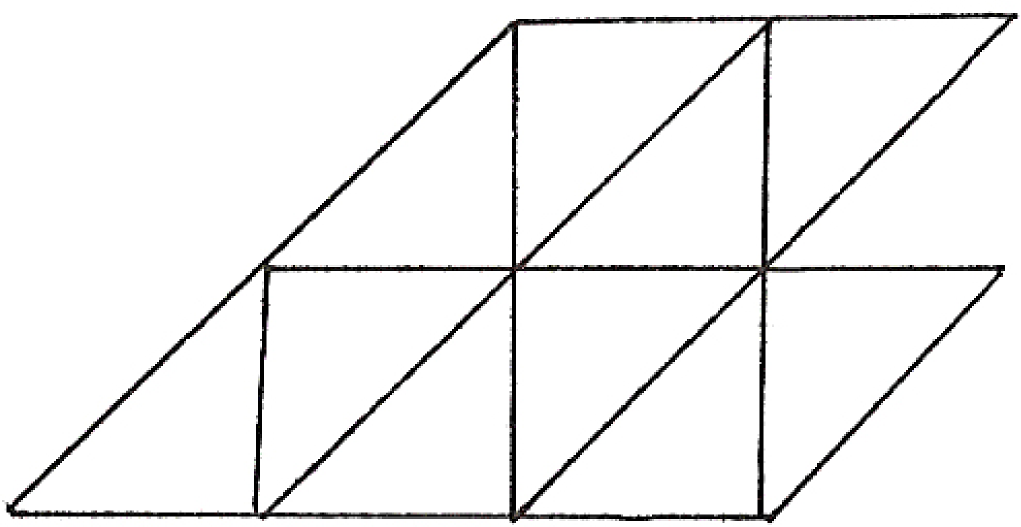


Figure 96

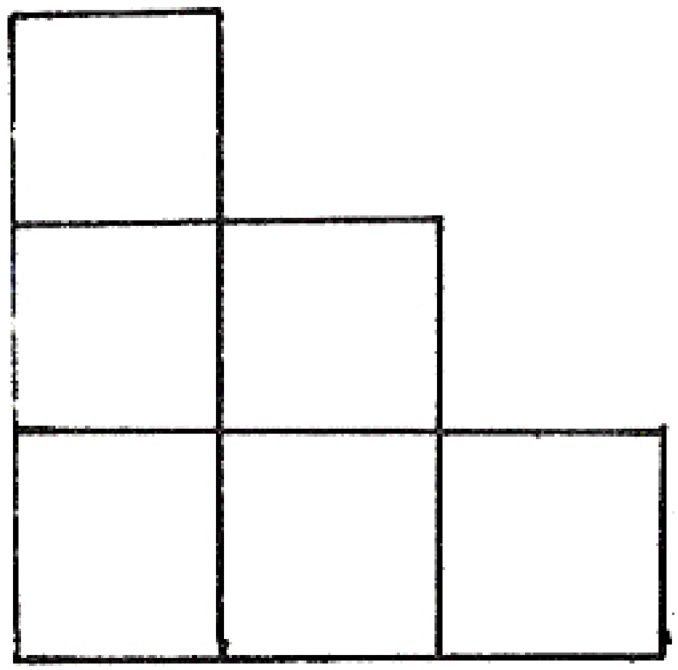


Figure 97

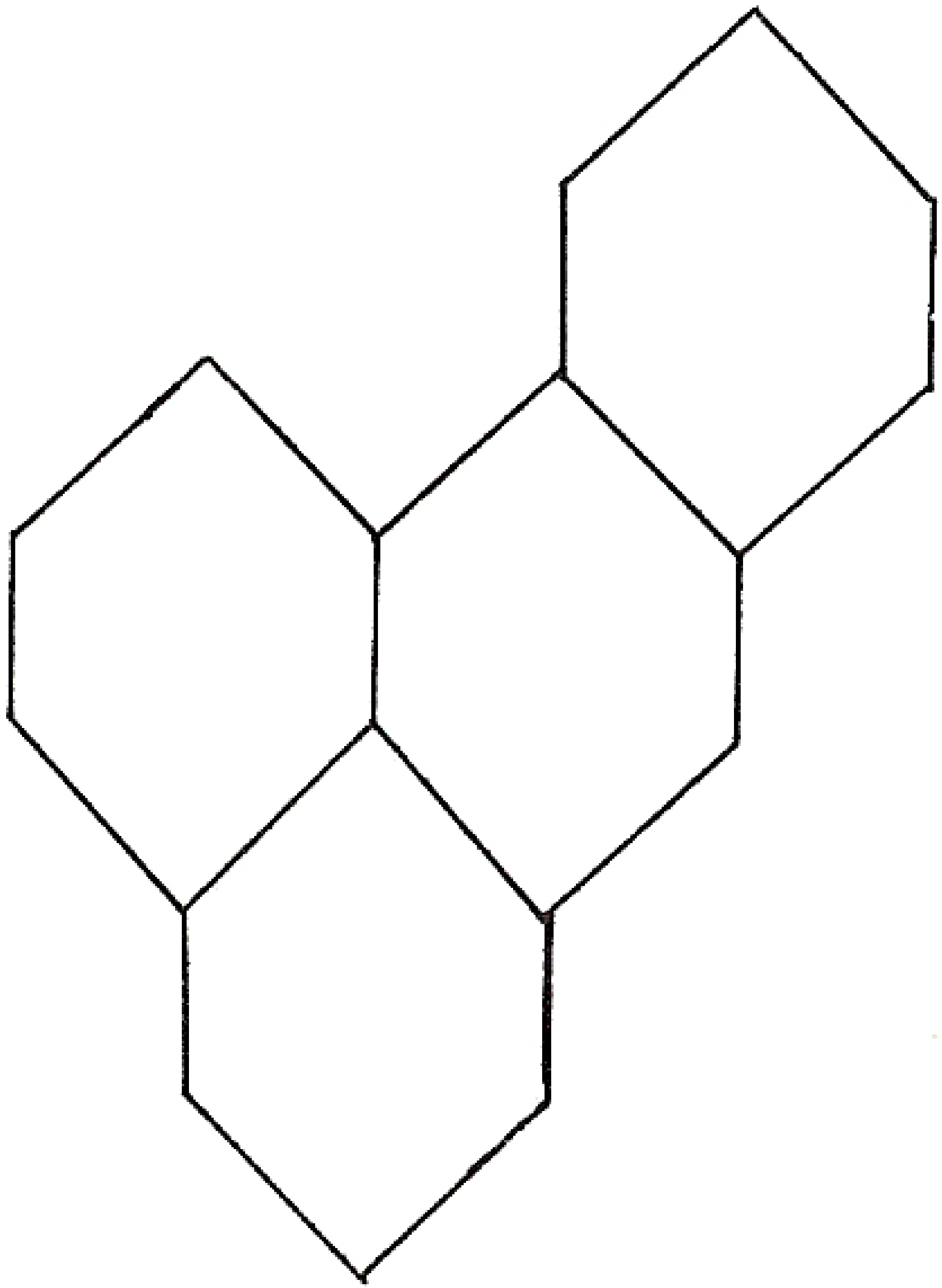
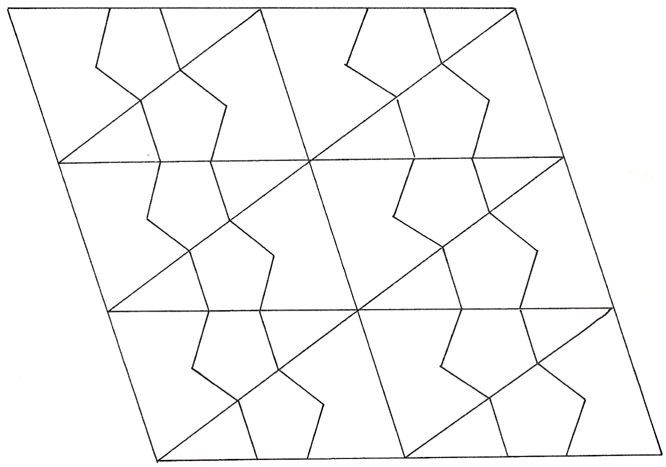
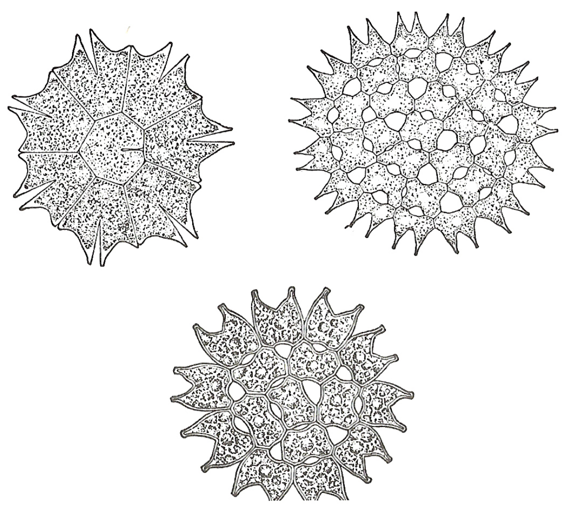


Figure 98

There do exist crystals, for example $MoAl_{12}$, that contain icosahedral elements within their component cells. But only a subgroup of the icosahedron's many symmetries is employed in the structure, and the icosahedron merely goes along for the ride. It is impossible for it to use its five-fold symmetry. The case of the icosahedral element within the $MoAl_{12}$ crystal can be compared to a set of triangular tiles, each with a pentagon pattern within. But the pentagon, although it might touch a side, would leave the pattern up to the simpler shape that it lived within.



(a) $MoAl_{12}$ crystal pattern



(b) Pediastrum species

Figure 99: Five Fold Symmetry Examples

Three different species of Pediastrum. (From Brown, The Plant Kingdom, Ginn, 1935 [Bro35]) Note that the two on the right have five-fold symmetry.

Five-fold symmetry does appear scattered among other symmetries in nature.

9.1 Growth and Generations of Stars

Below, we have three different patterns. Each has been produced by a different star following the same rule of growth. In one case, the pattern is that of squares, in other triangles and in the largest a strange pattern of over-lapping pentagons and five pointed stars.

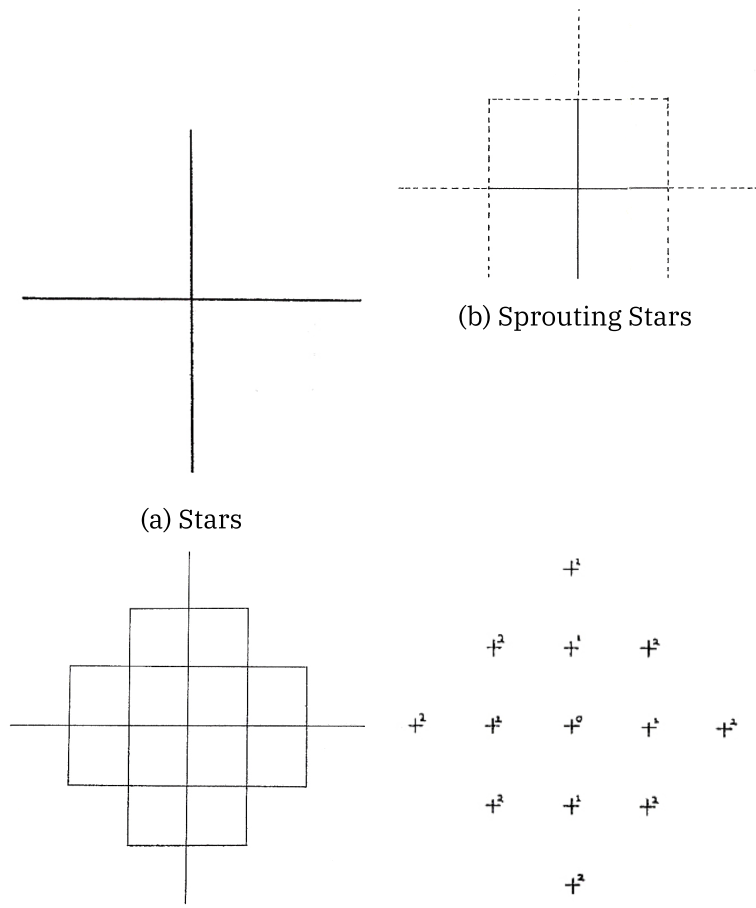
The rule followed is that the star sprouts other stars similar to itself at each of its end points. Each old end point must sprout before new ones sprout.

This is called recursive growth. In the first two cases, it produces a simple and uniform pattern which, as it grows, duplicates itself across the page.

The patterns to the right of the line patterns indicate at which generation the point was produced.

The regularity and homogeneity of the patterns of squares and triangles indicate the simplicity of growths that follow these symmetries. These are patterns of crystal growth—billions of identical molecules can be incorporated identically in these patterns.

In the case of the pattern with five-fold symmetry, there isn't uniformity. Different points have different patterns in their immediate neighborhoods. Instead of the pattern simply reproducing itself across the page, it becomes steadily more intricate.



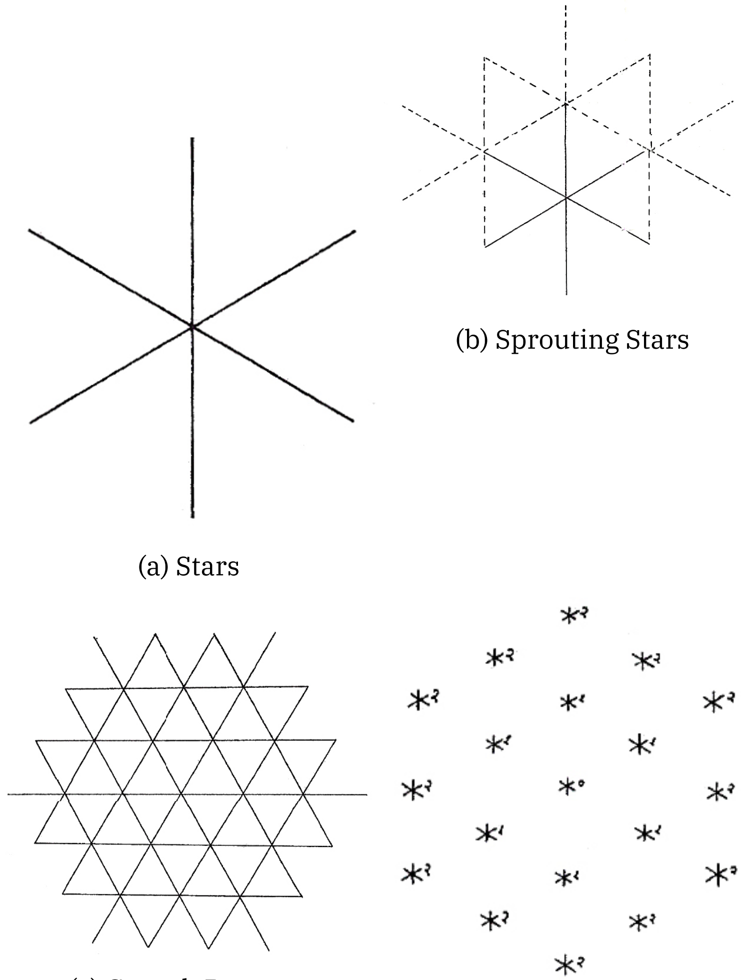
(a) Stars

(b) Sprouting Stars

(c) Growth Patterns

(d) Generations

Figure 100: Square Patterns



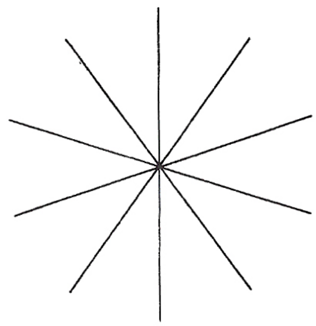
(a) Stars

(b) Sprouting Stars

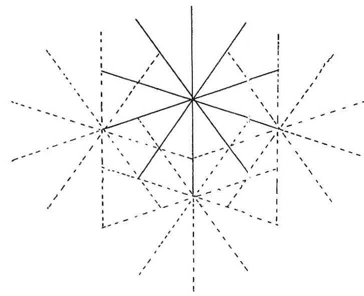
(c) Growth Patterns

(d) Generations

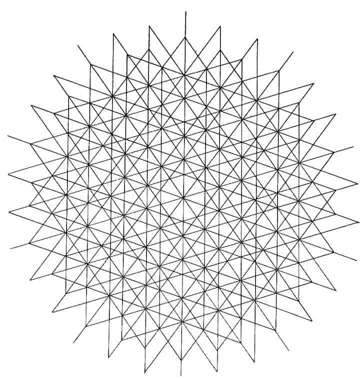
Figure 101: Triangle Patterns



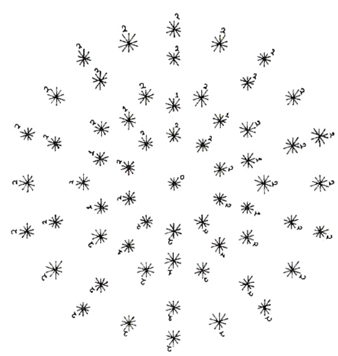
(a) Stars



(b) Sprouting Stars



(c) Growth Patterns



(d) Generations

Figure 102: Pentagon Patterns

10 Sections

If the lines of the 31-zone star followed no pattern, it would be possible to form $\frac{31 \times 30}{2 \times 1} = 465$ planes—each with a different orientation.

In our 31-zone system, the pairs of lines form only 121 different planes—this is because some of the pairs of lines lie on the same plane. Thus, our 31-zone star is singular.[1]

In any one 31-zone star there are:

1 See definition of singular star - Figure 21

15	R	sections
30	S	sections
6	T	sections
10	V	sections
30	X	sections
30	Y	sections

Table 10.1: Section types in 31 zone star

10.1 R Section

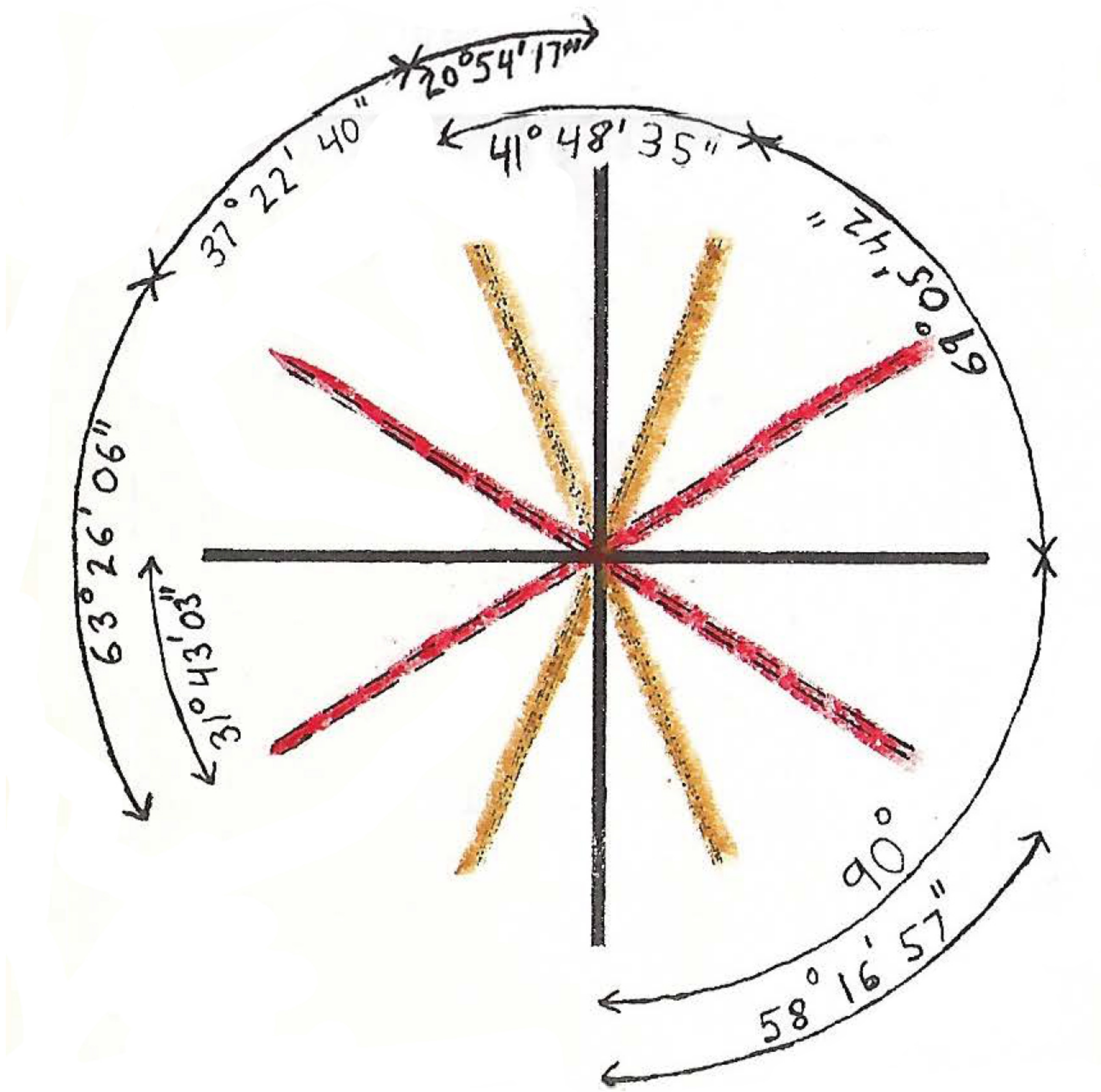


Figure 103: R Section Star

Figure 104: R Section Equators

Figure 105: One R Section Equator

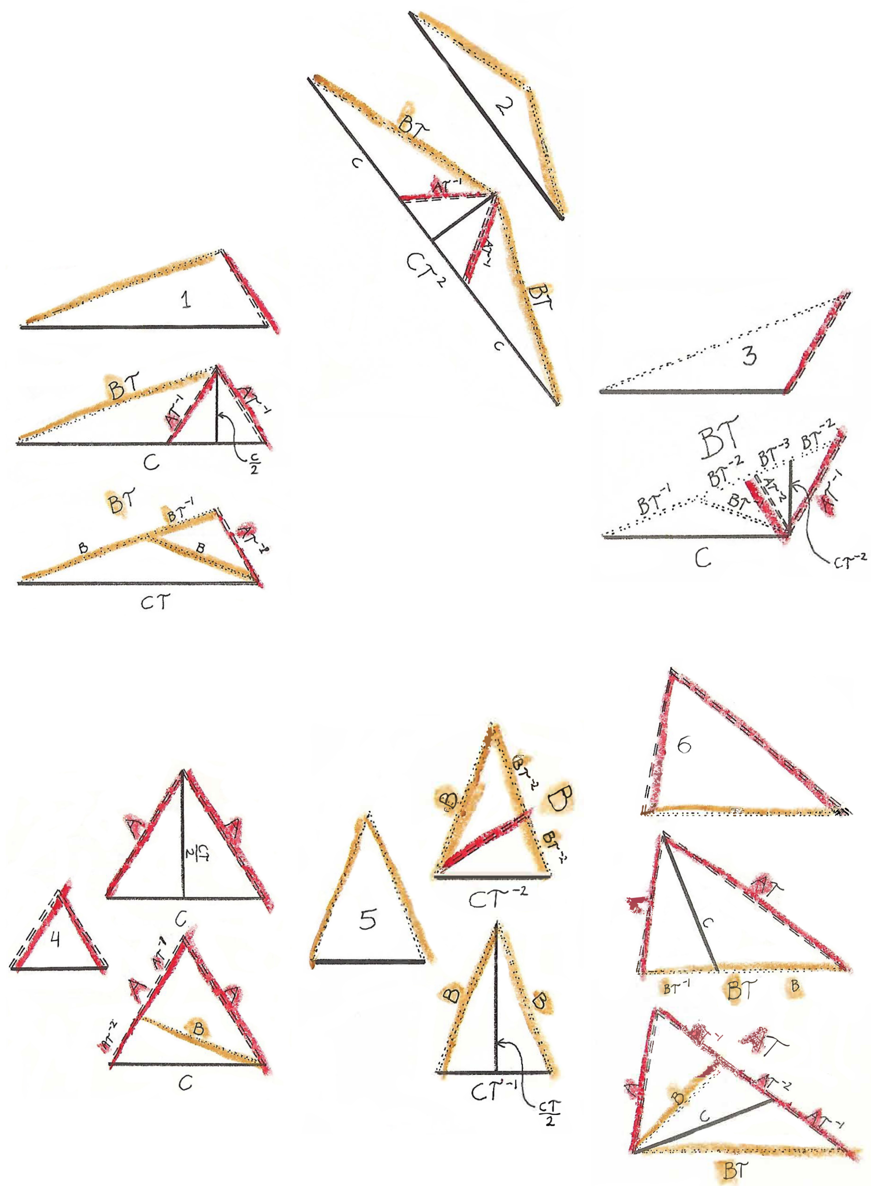


Figure 106: Species of R Section Triangles and Their Subdivisions 1–6

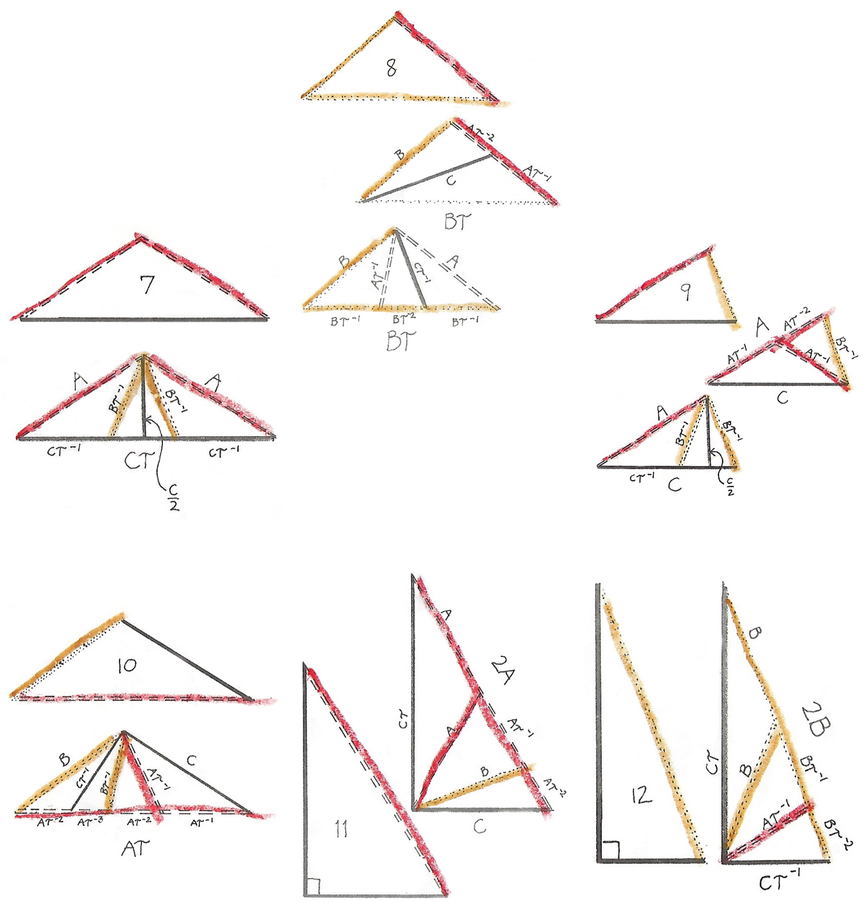


Figure 107: Species of R Section Triangles and Their Subdivisions 7–12

10.2 S Section

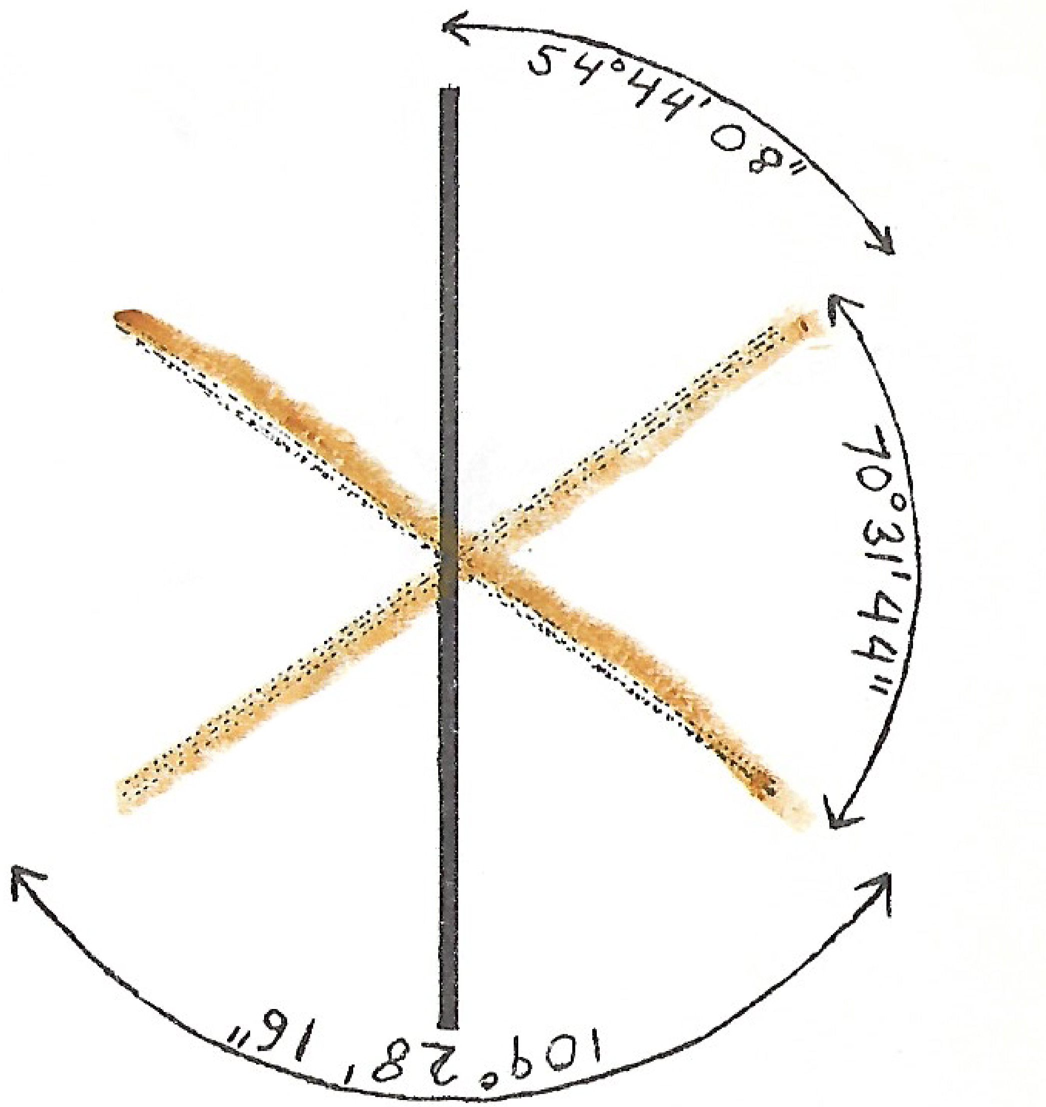


Figure 108: S Section

Figure 109: S Section Equators

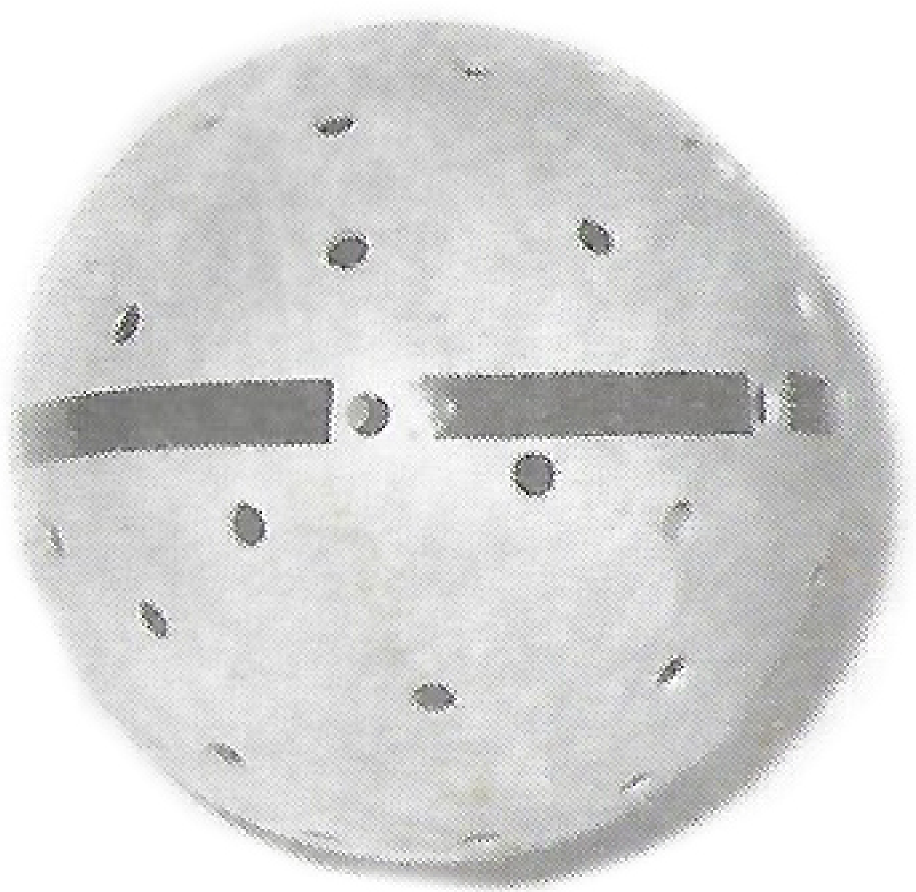


Figure 110: One S Section Equator

In the S section there is only one triangle possible:

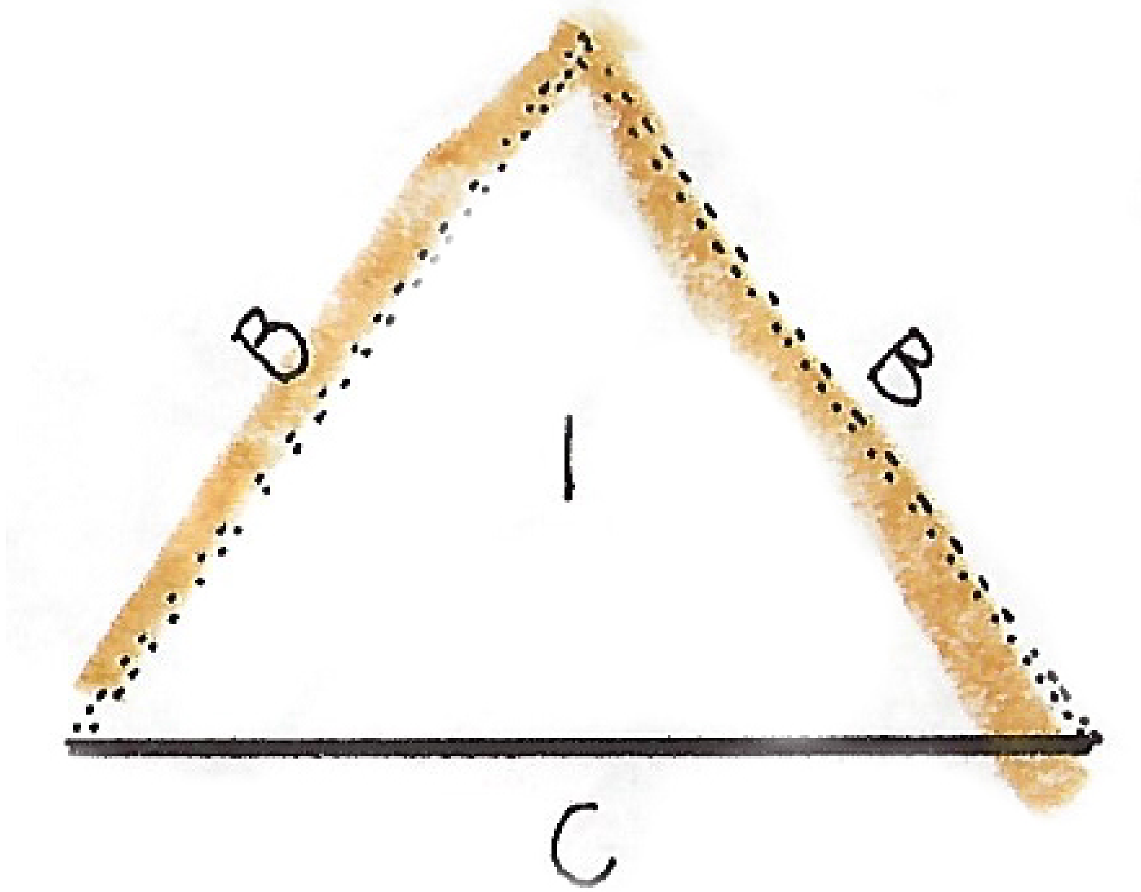


Figure 111: S Section Triangle

10.3 T Section

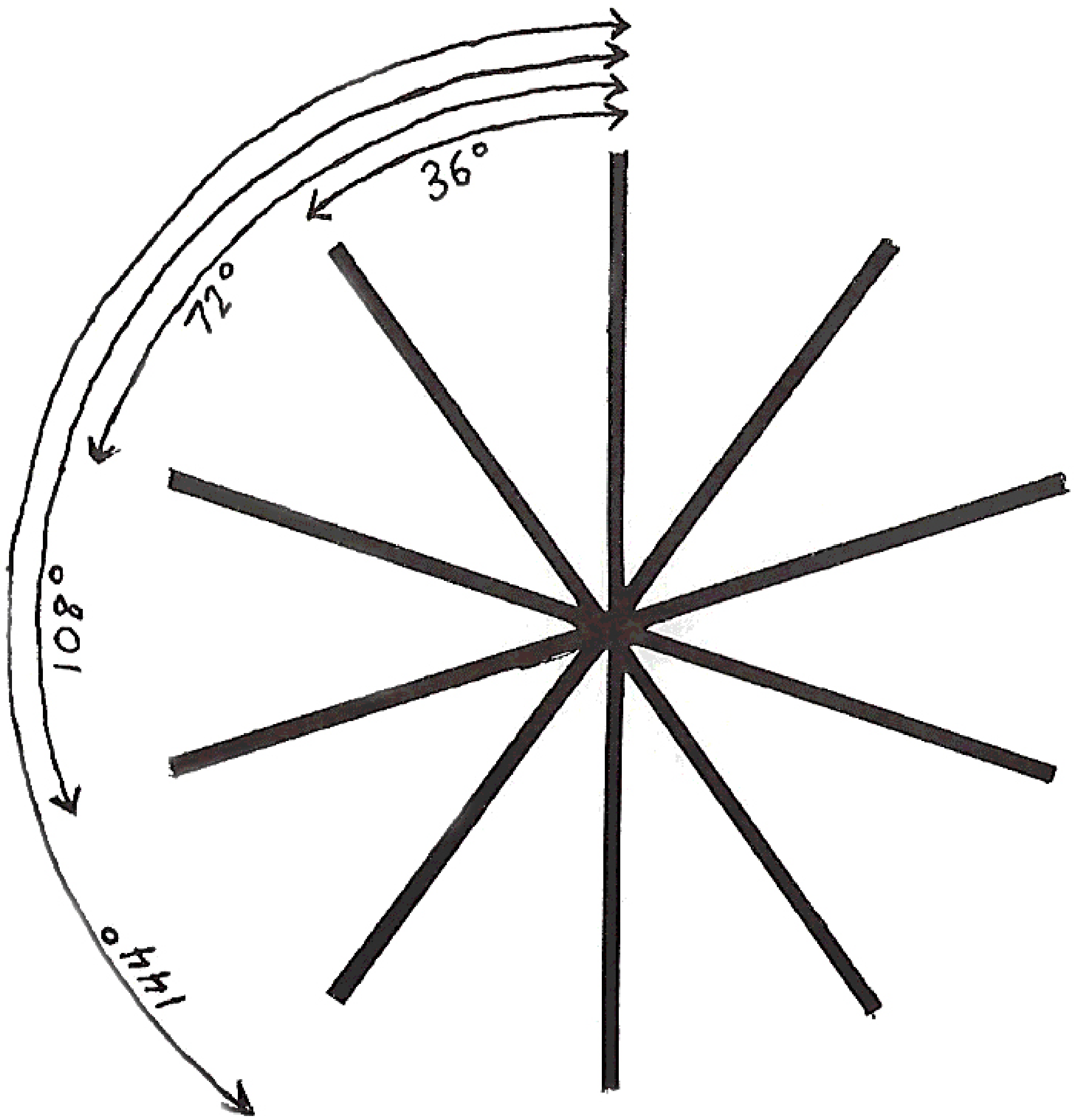


Figure 112: T Section

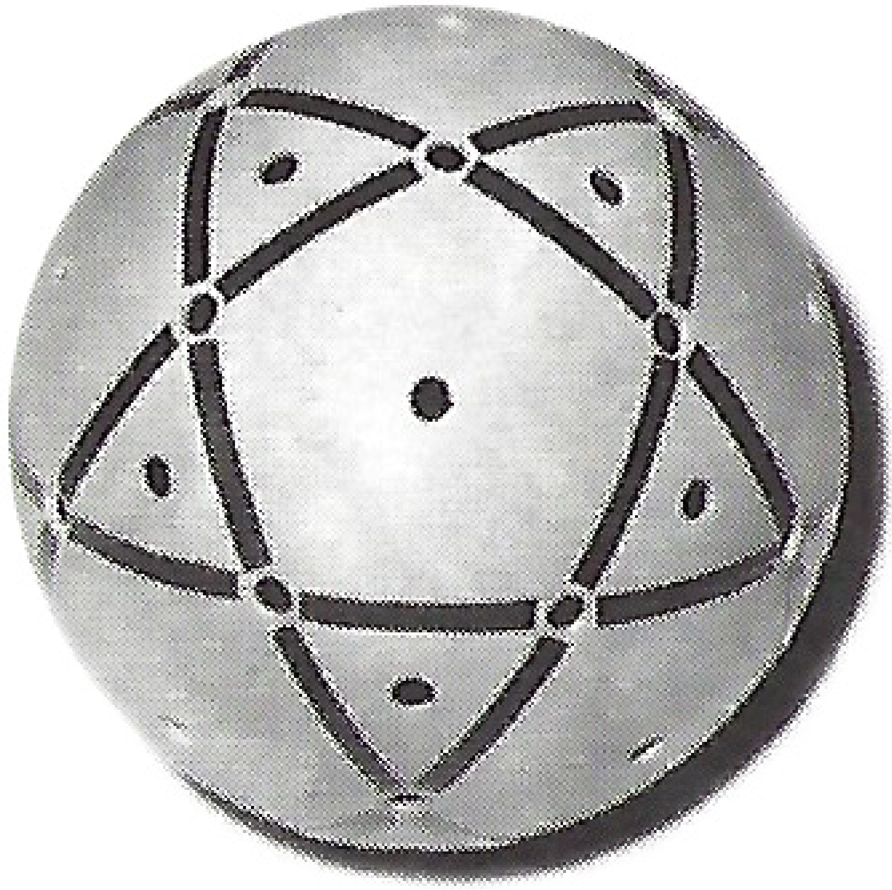


Figure 113: T Section Equators

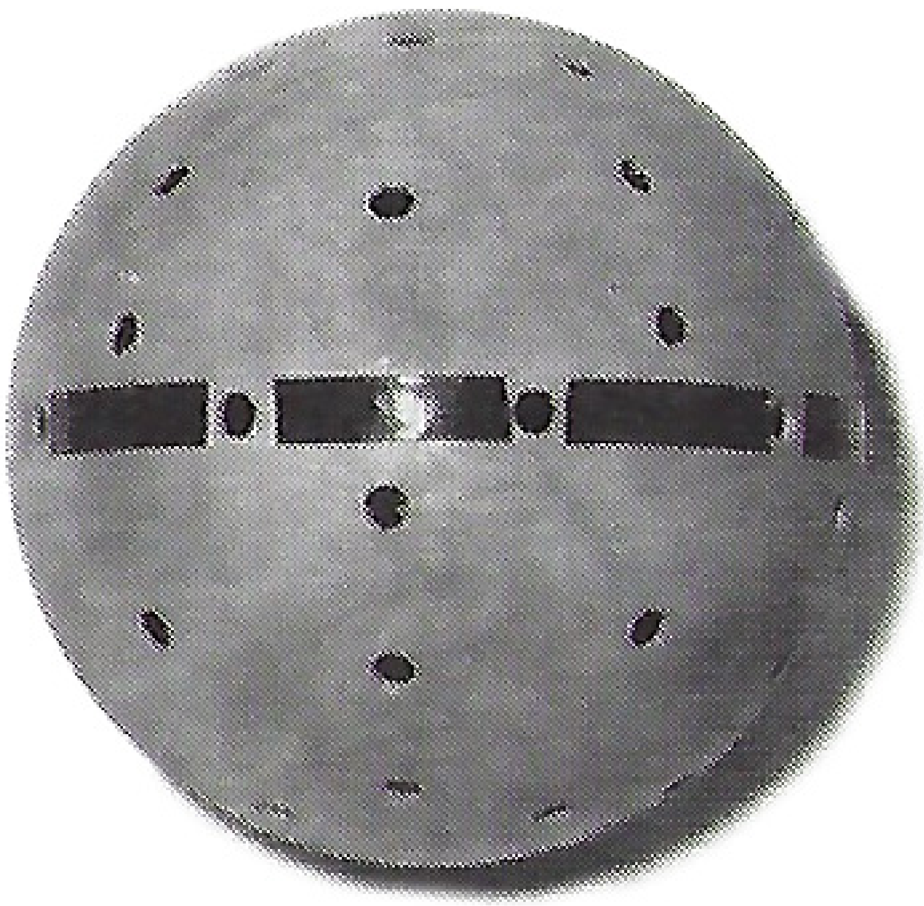


Figure 114: One T Section Equator

In the T section there are two kinds of triangles possible:

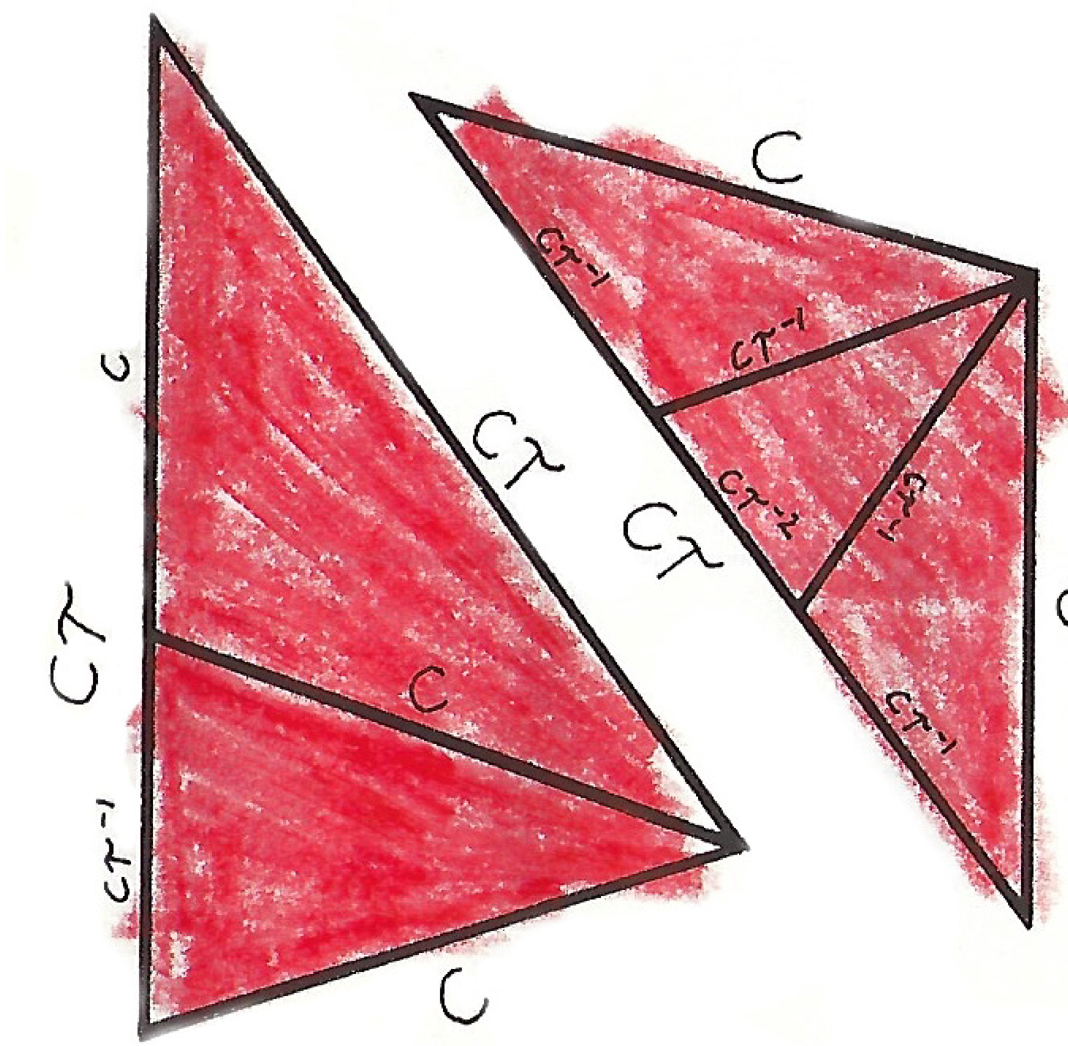


Figure 115: T Section Triangles

10.4 V Section

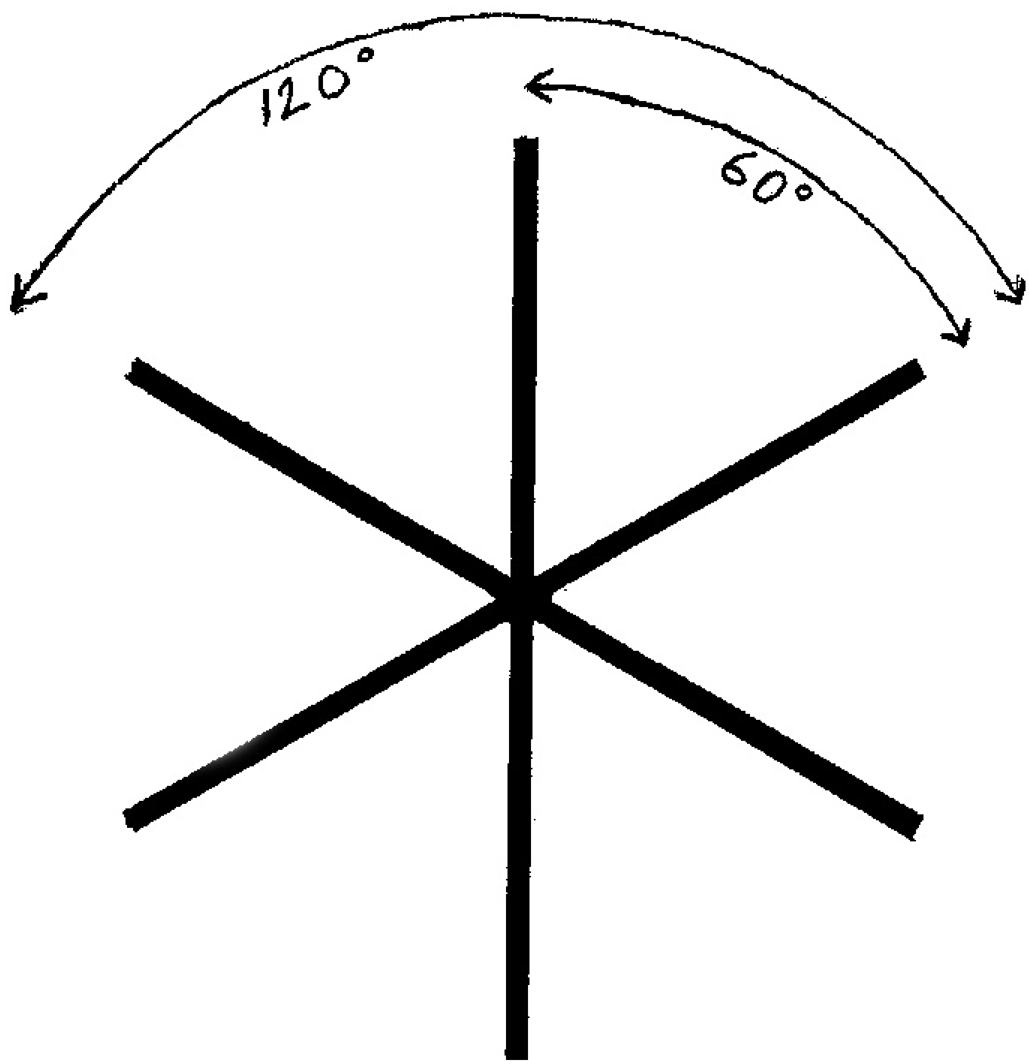


Figure 116: V Section

Figure 117: V Section Equators

Figure 118: V Section Equator

In the V section there are equilateral triangles:

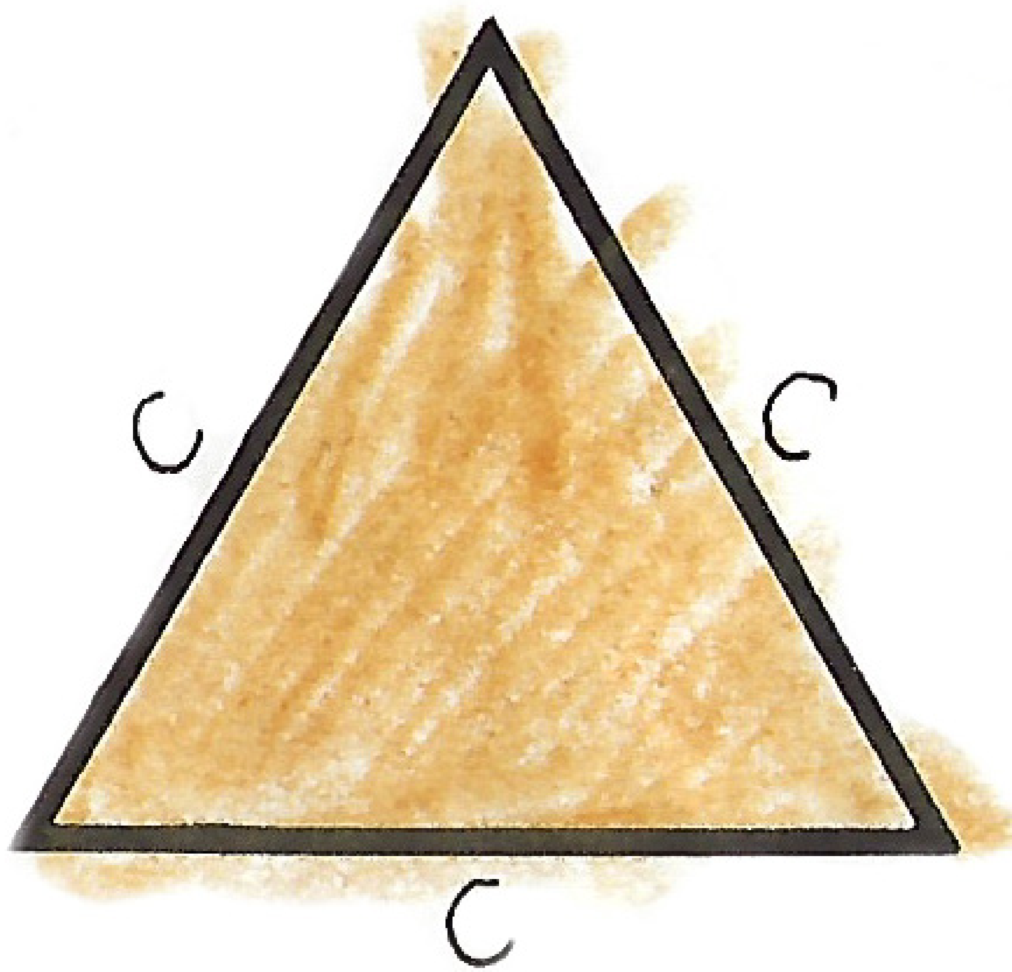


Figure 119: V Section Equilateral Triangle

10.5 X Section

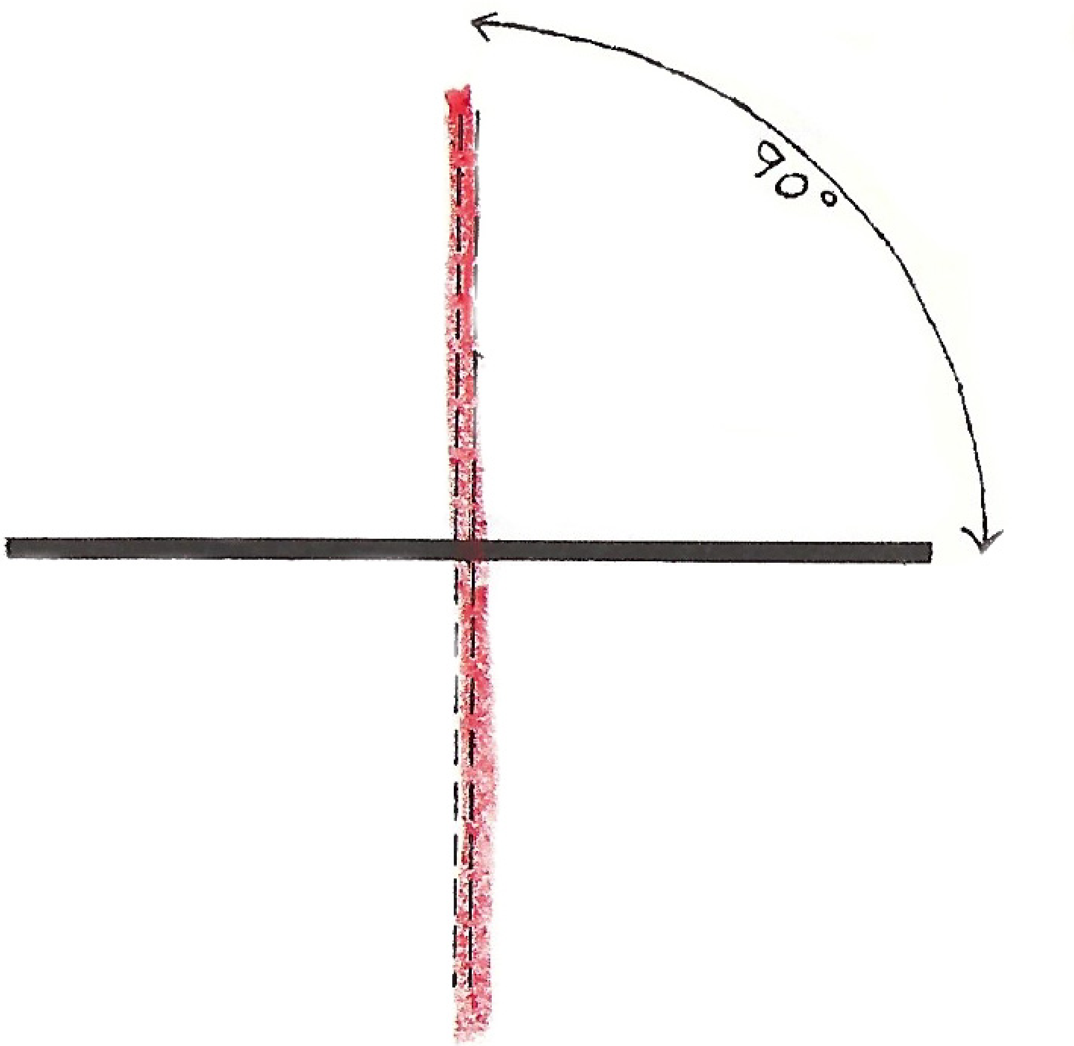


Figure 120: X Section

Figure 121: X Section Equators

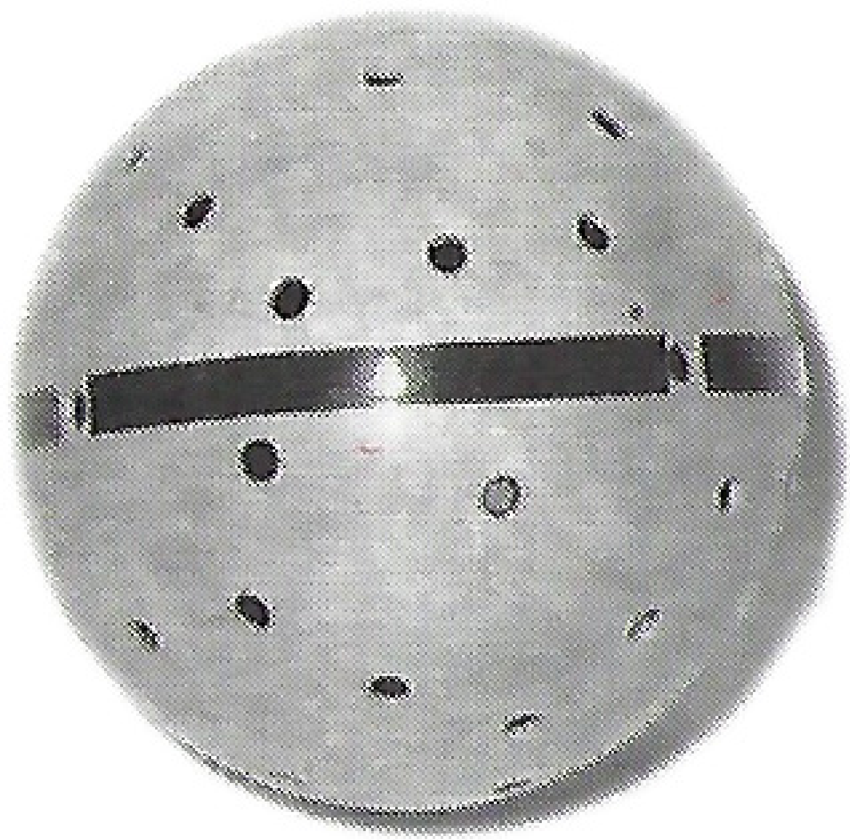


Figure 122: One X Section Equator

In the X section there is a rectangle:

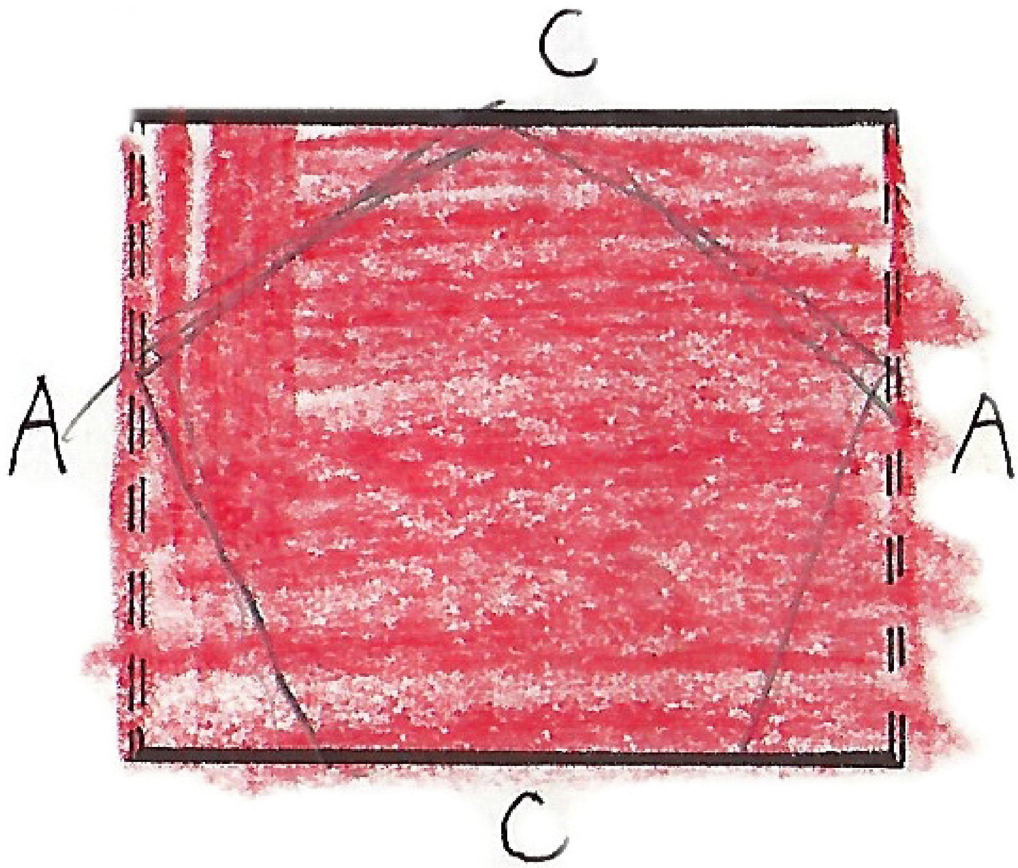


Figure 123: X Section Rectangle

10.6 Y Section

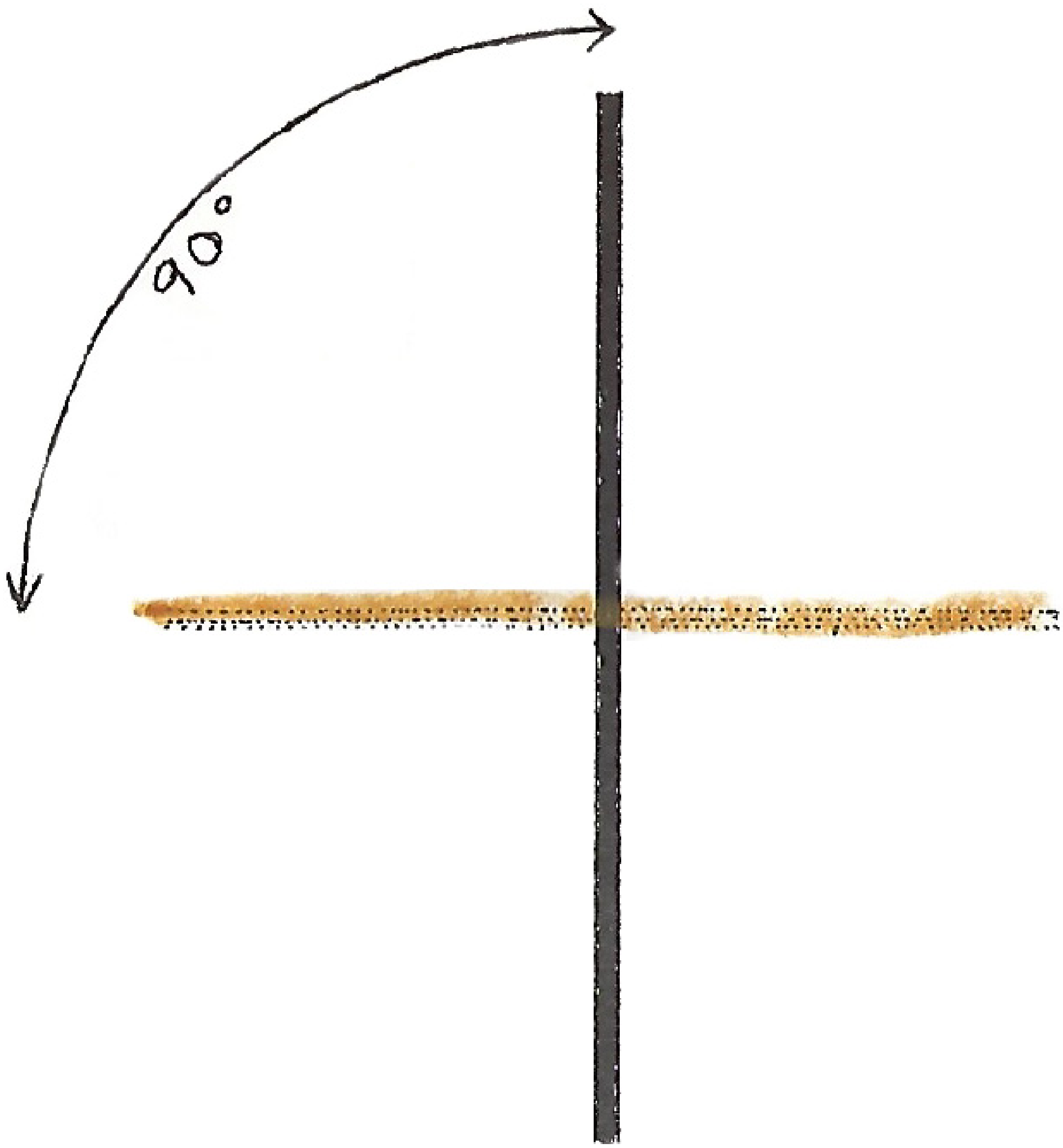


Figure 124: Y Section

Figure 125: Y Section Equators

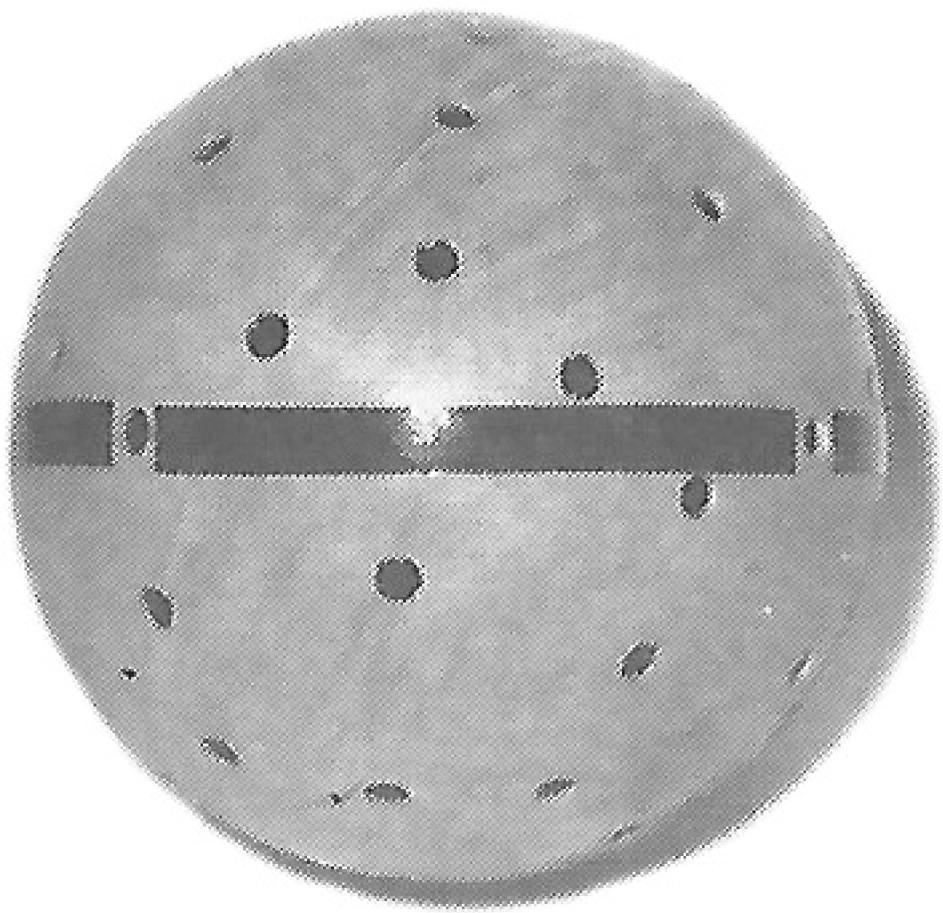


Figure 126: One Y Section Equator

In the Y *section* there is a rectangle:

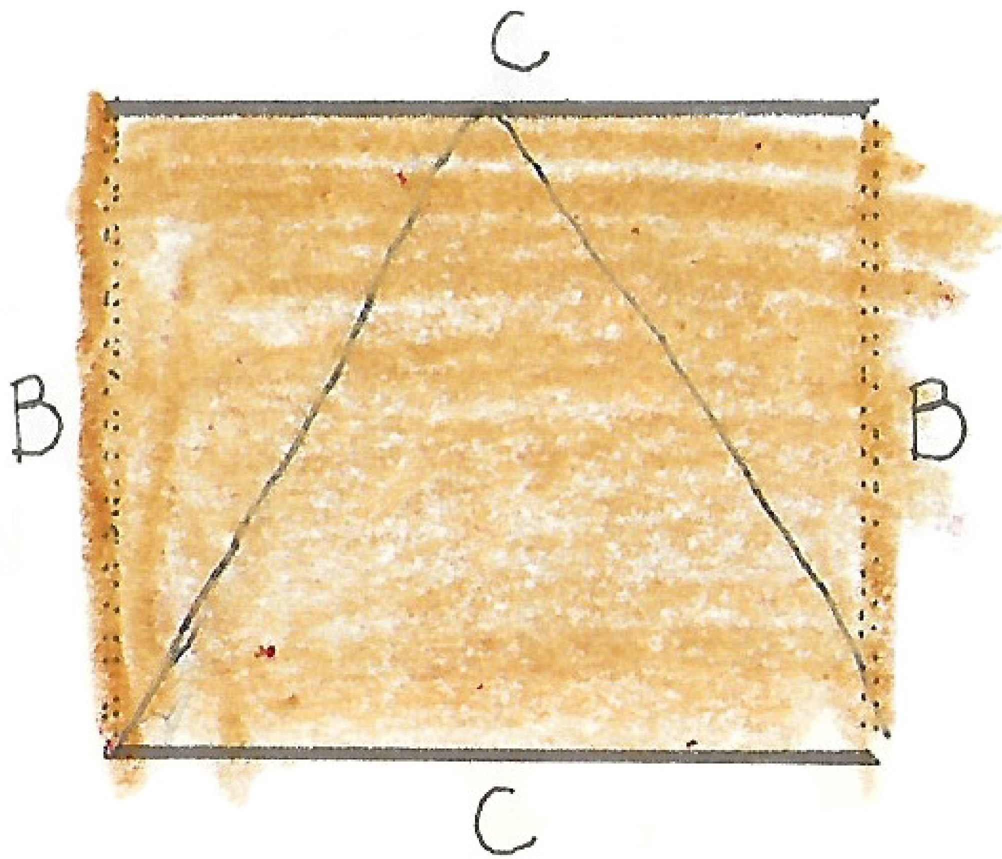


Figure 127: Y Section Rectangle

10.7 The Formation of Triangles

The R, S, T, and V sections contain triangles; the X and Y don't. They couldn't because they don't have enough lines. Every triangle has sides running in three different directions, but the X and Y have only two directions.

If we have three zones in one plane, then we can form a triangle. If it is a structure with only certain lengths of structural members, then we must have them proportioned correctly so that they begin and end at the vertices of these particular kinds of triangles. This is a more difficult problem. The A and B lines are unique in that stars formed of only A lines or B lines are non-singular. They define maximum numbers of planes—most uniformly distributed through space—with minimum numbers of lines. But they cannot triangulate themselves—mixed A and B lines do form triangles as can be seen in the triangles within the R sections. But it is the 15 C lines that serve as the triangulators in our star. In the T and V sections they form triangles with themselves while in the R and S sections the very relationship between the lengths of the A and B lines was determined by choosing the same C lines as a base for two different triangles—one with A lines and one with B lines.

10.8 A Star Problem

An interesting problem for the designer would be to construct a star in which every plane defined in the system by two lines could be triangulated by other lines in the system. Or, to arrange lines in space, so there were never only two lines in one plane. This is impossible—as you add more lines to triangulate a plane, the new lines define new planes with old lines, and the star needs even more new lines to form triangles. That the X and Y sections in the thirty-one zone star are unable to triangulate themselves cannot be avoided. A proof of this assertion can be derived from the necessity of all convex polyhedra to have some faces with fewer than six sides.

We have shown all the triangles that can be formed with our system. We have not illustrated all the convex polygons—those with an even number of sides are straight forward. The existence of irregular pentagons, septagons, nonagons and eleven sided figures has not been investigated.

There are a finite number of classes of such an angle similar convex polygons. If one does not insist that the polygons be convex, then there are infinite numbers of such polygons.

In three dimensions, the smallest convex polyhedron is the tetrahedron. A tetrahedron has 4 triangular sides. The stock of possible triangles to form tetrahedra is those we have shown in the R, S, T and V sections. These triangles must fit together along the proper planes to form tetrahedra within our system. For instance, in the V sections there are equilateral triangles, but the dihedral angles between V sections do not allow us to form a regular tetrahedron in our system.

11 The Divine Proportion

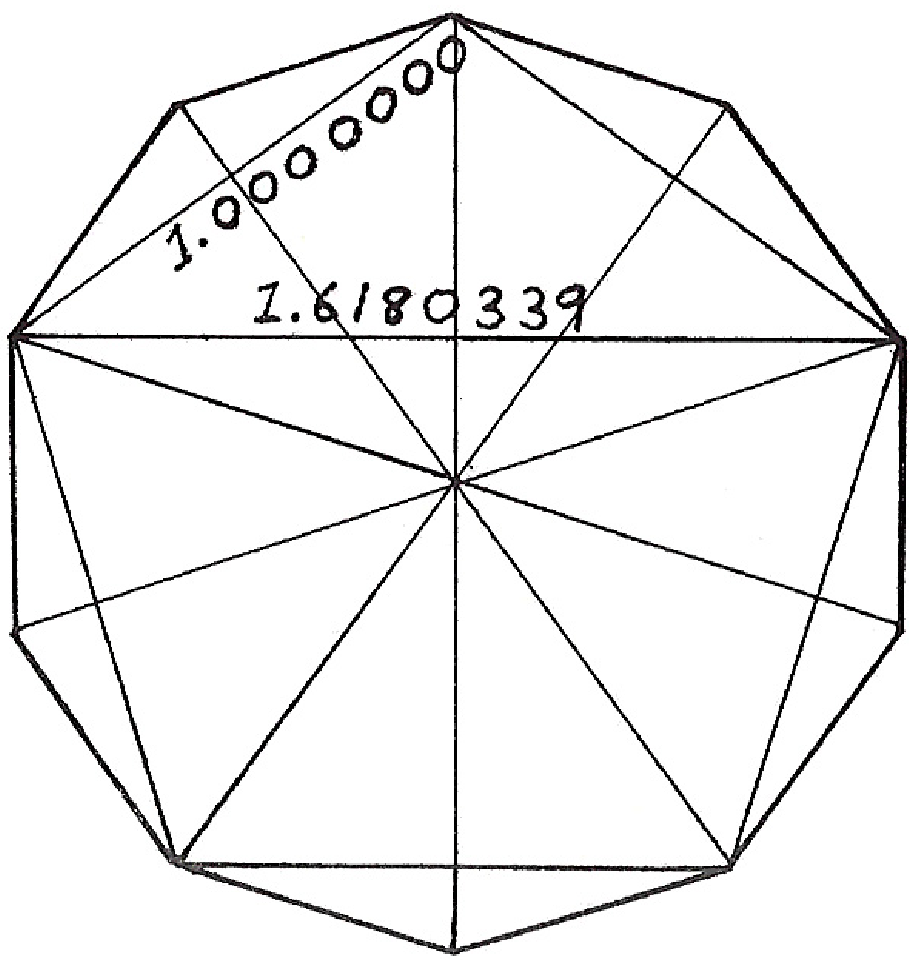


Figure 128: Divine Proportion

THE DIVINE PROPORTION, or 1.6180339 … is the relationship between the diagonal of a pentagon to the edge. This accounts for its ubiquitous appearance in our structures with five-fold symmetry, for these structures are continuously forming pentagons. Again and again components can be found as diagonals or sides of pentagons made with other components—or both. But this isn't really reason enough. It is hard to think of any of the numerous divine proportion relationships as cause for others. Rather, they all seem symptoms of a deeper set of relationships. And our fastening on the divine proportion and the Fibonacci numbers[1] seems peculiar when we consider, for instance that the square root of 2 appears again and again in grids of squares.

1 See Section 11.2

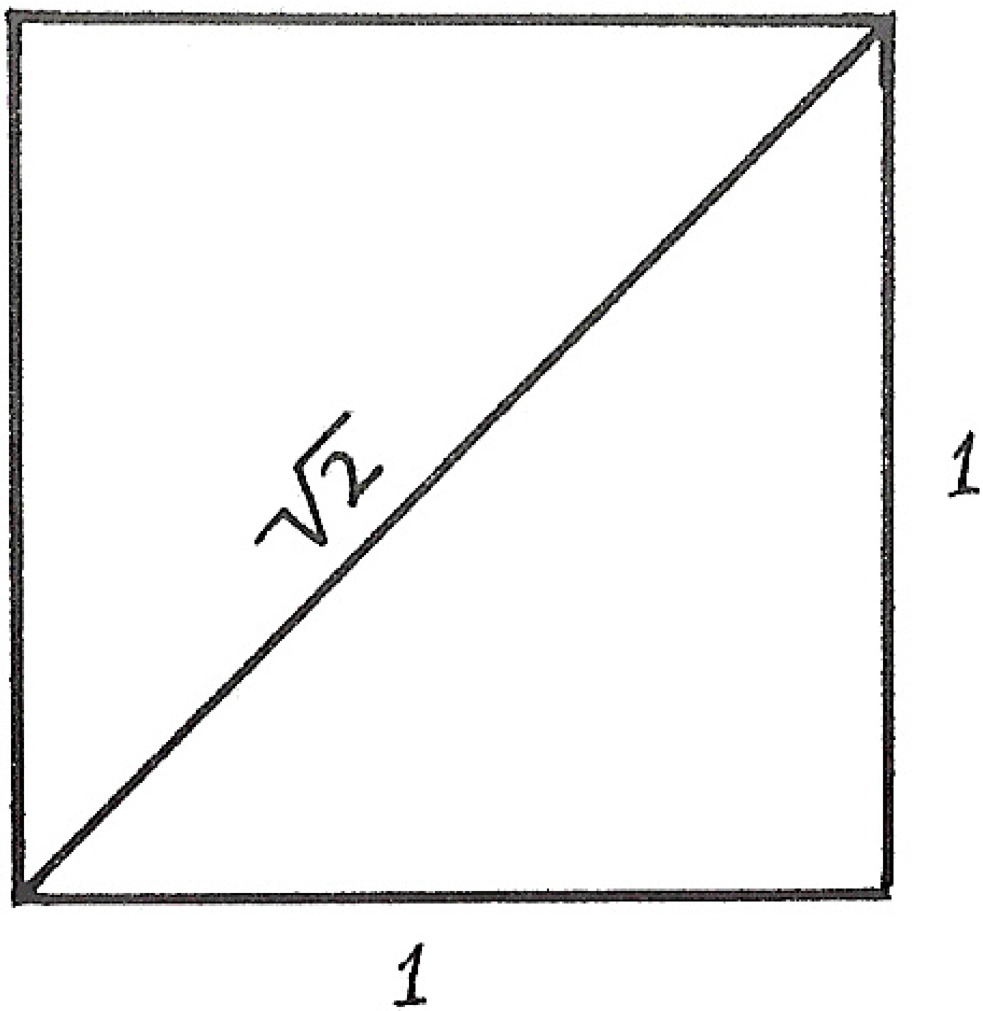


Figure 129: Square root of 2

And that the square root of 2, an irrational number, is approached by a simple sequence of fractions.

$$\frac{1}{1}, \frac{3}{2}, \frac{7}{5}, \frac{41}{29}, \frac{99}{70} \tag{11.1}$$

Each denominator is the sum of the numerator and the denominator of the preceding fraction. Each numerator is the sum of its own denominator and the preceding one. Or, if one uses a pattern of seven lines, there are still other relationships and their powers that appear and reappear.

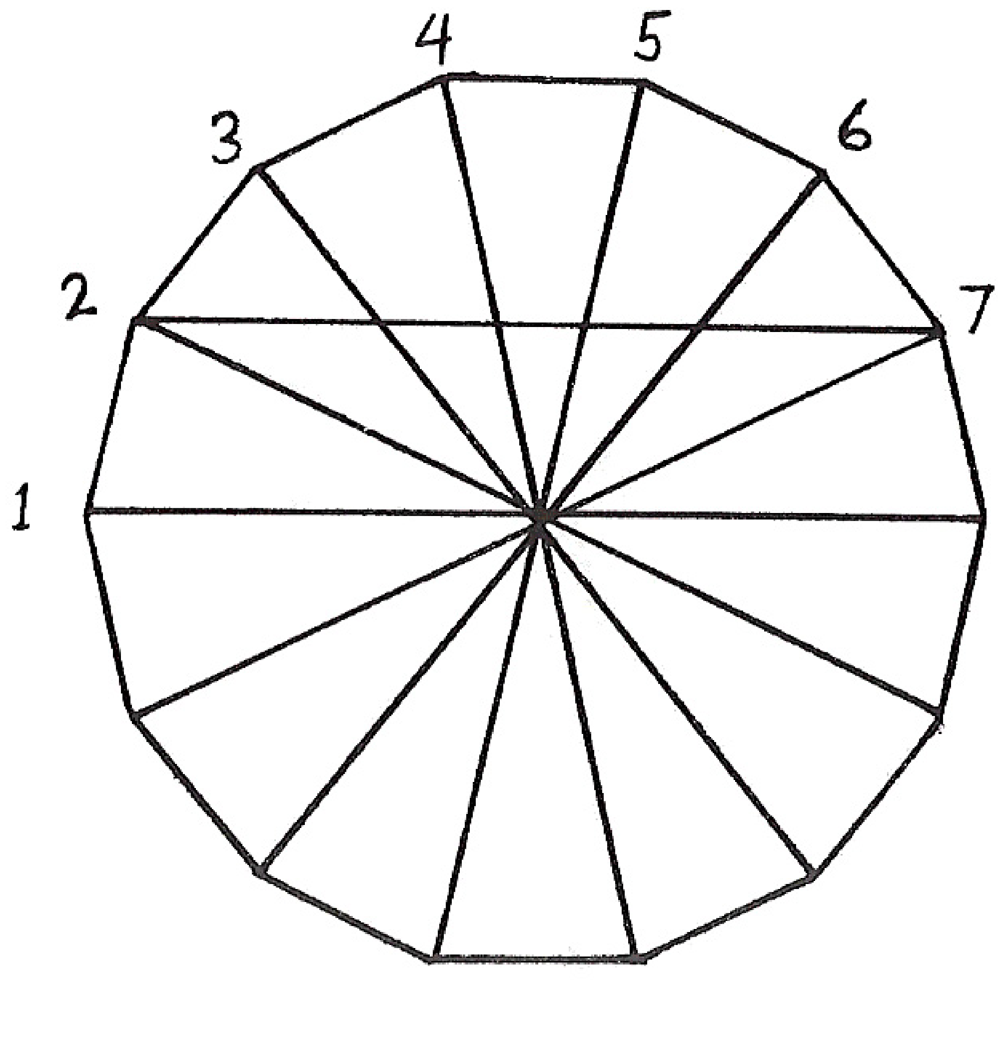


Figure 130: 7-Zone Star

In the 7-zone star the relationship between the lengths of the radii and the line $\overline{45}$ can be approached by the relationship between successive integers in a sequence formed in much the same way as the divine proportion's Fibonacci series. The rule for forming the sequence is that in the sequence any integers S_n:

$$S_n = 2S_{n-1} + S_{n-2} - S_{n-3} \tag{11.2}$$

or, for example,

$$283 = 2(126) + 56 - 25 \tag{11.3}$$

The first terms in the sequence are...1, 2, 5, 11, 25, 56, 126, 283, 636, 1429, 3211, 7215, 16212, 36428, 81853

$$\frac{11}{5} = 2.20000 \tag{11.4}$$

$$\frac{283}{126} = 2.24698 \tag{11.5}$$

$$\frac{7215}{3211} = 2.24696 \tag{11.6}$$

$$\frac{81853}{36428} = 2.24698 \tag{11.7}$$

Calculating the value from 10 place trigonometry tables

$$\frac{radius}{\overline{45}} = 2.24701 \tag{11.8}$$

The relationship between the radius and the line $\overline{27}$ is approached by the ratio between adjacent terms in a sequence where:

$$S_n = S_{n-1} + 2S_{n-2} - S_{n-3} \tag{11.9}$$

The more you examine properties of objects and phenomena, the more you find yourself presented with a few terms, usually simple, from a long series of terms. Often you cannot touch the terms which are farther or lower in the series, but you can define properties which they have. One gets the feeling of living in a container - one of an infinite number - to which are shunted objects and phenomena which have passed through one filter but can't pass through another; a great process like that which takes place in a gravel yard, only we are unable to see gravel other than that of our own size but sense that it exists in endless different piles beyond - everything from sand to piles of planet sized boulders.

T expressed as a continued fraction.

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}} \tag{11.10}$$

We can truncate the fraction anywhere and compute its value. The farther we carry it the closer it approximates the exact value;

$$T = \frac{\sqrt{5} + 1}{2} \tag{11.11}$$

11.1 Tau Power Series

T^{-3}	=	0.2360680
T^{-2}	=	0.3819660
T^{-1}	=	0.6180340
T^{0}	=	1.0000000
T^{1}	=	1.6180340
T^{2}	=	2.6180340
T^{3}	=	4.2360680
T^{4}	=	6.8541020
T^{5}	=	11.0901699
T^{6}	=	17.9442719
T^{7}	=	29.0344418

Table 11.1: Tau Power Series

$$T^n = T^{n-1} + T^{n-2} \tag{11.12}$$

11.2 Pattern and Rules

If you begin with any two numbers and follow the rule, each term equals the sum of the two preceding terms, the ratio between consecutive terms approaches the divine proportion.

The first two terms of the Fibonacci numbers are 1, 1, and their proportion $\frac{1}{1}$ is a long way from T, but $\frac{F_n}{F_{n-1}}$ quickly approaches T.

$$\frac{F_7}{F_6} = 1.6250 \tag{11.13}$$

$$\frac{F_{12}}{F_{11}} = 1.6180 \tag{11.14}$$

Accompanying many simple polygons and patterns of polygons are series such as the Fibonacci, where the relationships between different terms approach the precise geometric relationships. The rules for forming the accompanying series are then clues for examining the structure of the pattern. And the relationships repeated in the pattern are clues for rules which give the process to form the pattern.

The perfection of the geometric form seems fragile. If we demand perfection to 7 decimal places, the thickness of a line spoils our form.

But the rules for the formation of the series which accompany the pattern are sturdy and simple. If mistakes are made in a sequence formed by the rules, the sequence heals itself after a few generations to again approach the precise form. This is seen in how quickly the Fibonacci series approaches T after its clumsy beginning.

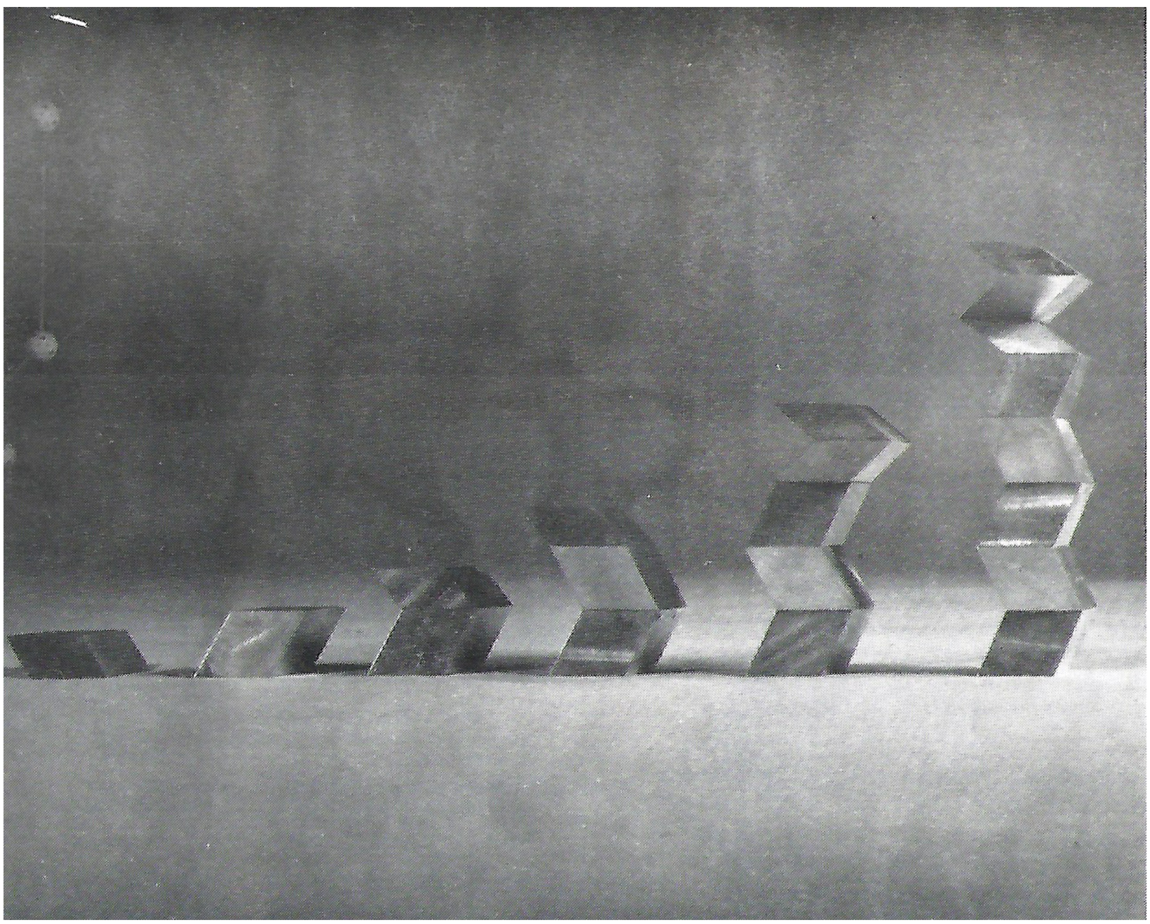


Figure 131

Stacks of six zone acute and obtuse cells. The widths of an acute cell and an obtuse cell are in the divine proportion. Therefore, a series of stacks can be built with each stack 1.6180339... times as tall as the one before it. The rule is that each stack is made by placing the two stacks that precede it on top of each other.

There are also numerous relationships involving the divine proportion among the altitudes of the parallelepiped cells formed with "B" lines.

11.3 The Fibonacci Numbers

F_1	1	F_{11}	89	F_{21}	10946	F_{31}	1346269
F_2	1	F_{12}	144	F_{22}	17711	F_{32}	2178309
F_3	2	F_{13}	233	F_{23}	28657	F_{33}	3524578
F_4	3	F_{14}	377	F_{24}	46368	F_{34}	5702887
F_5	5	F_{15}	610	F_{25}	75025	F_{35}	9227465
F_6	8	F_{16}	987	F_{26}	121393	F_{36}	14930352
F_7	13	F_{17}	1597	F_{27}	196418	F_{37}	24157817
F_8	21	F_{18}	2584	F_{28}	317811	F_{38}	39088169
F_9	34	F_{19}	4181	F_{29}	514229	F_{39}	63245986
F_{10}	55	F_{20}	6765	F_{30}	832040	F_{40}	102334155

Table 11.2: Fibonacci Numbers

$$F_n = F_{n-1} + F_{n-2} \tag{11.15}$$

12 Coherence Proofs

In a structural system or any pattern the question arises; what is the pattern made of? What are the relationships between different elements of the pattern?

In a checkerboard pattern such as that shown in figure 100 all distances between neighboring intersections are the same. And the distance between two intersections of any line is simply a multiple of this base distance. This base distance then naturally becomes the unit for building the pattern.

In our pattern created by the star with five-fold symmetry, the situation is different. There are many different lengths between intersections. If the growth patterns follow simple rules such as those followed in forming the two-dimensional pattern of figure 102, then all distances between intersections can be expressed as simple sums of components whose lengths are equal to powers of the divine proportion times some constant. This is also true in three dimensions - the A and B lines of the 31-zone star forming a growth similar to our 2-dimensional growth intersect each other at points where the distance between any two intersections on an A line equals

$$s_1 AT^{r_1} + s_2 AT^{r_2} + \ldots\ s_n AT^{r_n} \tag{12.1}$$

and the distance between two intersections on a B line equals a polynomial

$$q_1 BT^{t_1} + q_2 BT^{t_2} + \ldots\ q_n BT^{t_n} \tag{12.2}$$

The building blocks for our system are then a series of lengths related by the divine proportion. An A series, a B series and a C series—each of slightly different lengths.[1]

1 See Critical Constants 7 and Hardware 15.

12.1 Two Dimensions

12.1.1 Definition 1

We call any particular polynomial of the form

$$s_1 kT^{r_1} + s_2 kT^{r_2} + \dots\ s_n kT^{r_n} = f_i(kT) \tag{12.3}$$

s_i and r_i are integers.

We call the class of all such polynomials $F(kT)$ then;

$$f_i(kT) + f_j(kT) \in F(kT) \tag{12.4}$$

$$f_i(kT) - f_j(kT) \in F(kT) \tag{12.5}$$

$$(T^n) f_i(kT) \in F(kT) \tag{12.6}$$

We are interested in the distances between intersections along the lines of certain sets of patterns.

12.1.2 Definition 2

A pattern is a number of extended lines which point in five different directions in one plane with at least one line pointing in each direction.

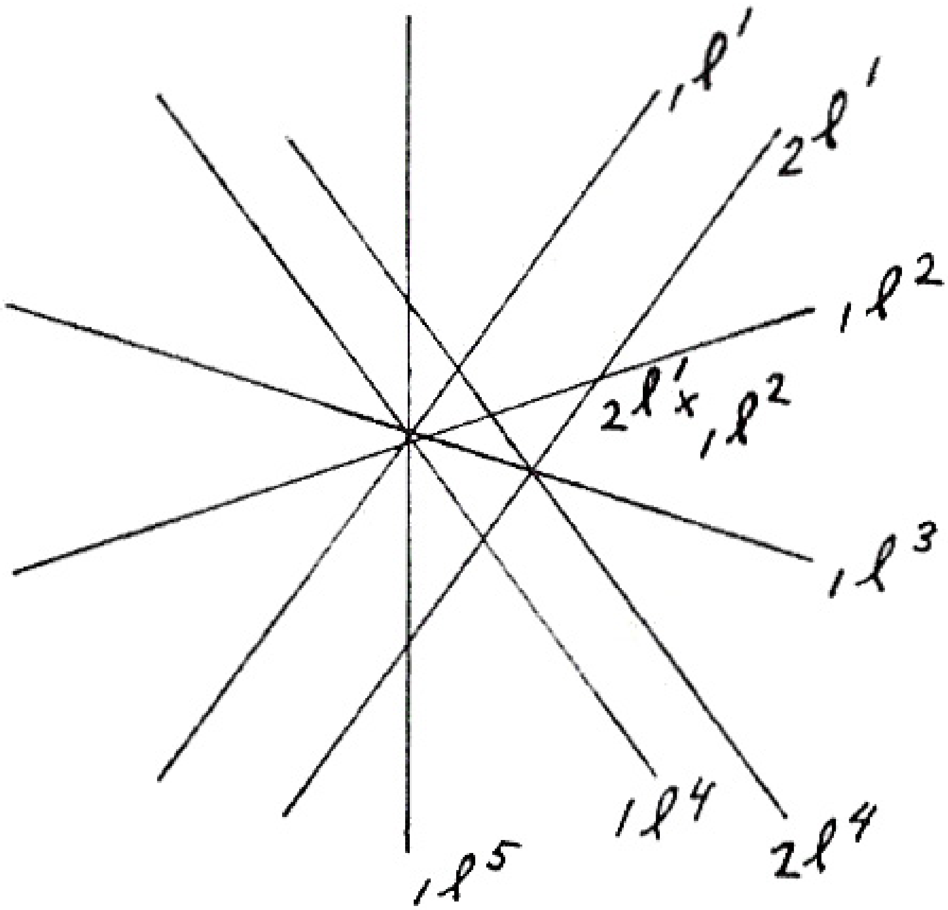


Figure 132

12.1.3 Definition 3

The lines are labeled;

$$i^{l^n}$$
$$n = 1, 2, 3, 4, 5$$
$$i = 1, 2, ...$$

n names the direction - i names the particular line pointing in that direction. The angles between lines are;

$$\frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}$$

12.1.4 Definition 4

An intersection is named by any two of the lines which intersect there, such as

$$j^{l^n} \times k^{l^m} \tag{12.7}$$

12.1.5 Definition 5

If on a line all intervals between intersections are equal to different $f_i(kT)$, then the intersections "fit" each other.

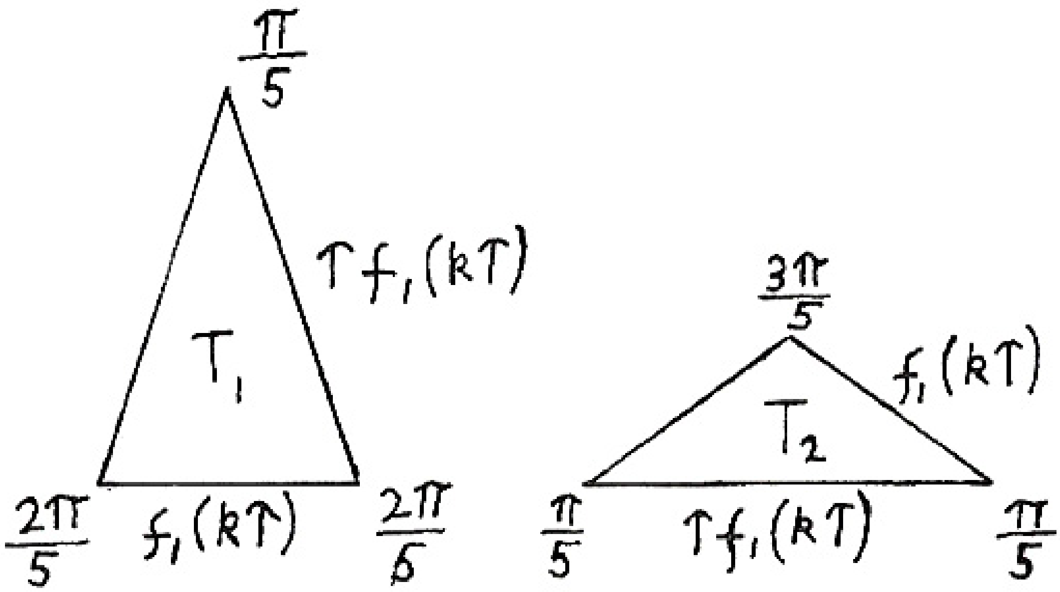
An intersection fits a line if it fits all intersections on that line.

A line g^{l^p} fits another line f^{l^q} if the intersection $g^{l^p} \times f^{l^q}$ fits all the intersections of f^{l^q}.

A pattern fits if all the intersections on all the lines fit.

Lemma 12.1.1. *If on a line l^n an intersection I_n fits an intersection I_m then I_n fits all I_o where I_o fits I_m.*

Lemma 12.1.2. *Any i^{l^m}, j^{l^n}, h^{l^p}, $m \neq n, n \neq p$, p form a triangle similar to T_1 or T_2*



12.1.6 Definition 6

These triangles are called the Golden triangles because their sides are in the divine proportion.

Lemma 12.1.3. *If the two vertices of a Golden Triangle fit each other then all vertices fit each other.*

Lemma 12.1.4. *If a line j^{l^m} fits a line h^{l^n} and if all l^n fit some line i^{l^m} and if i^{l^m} fits all l^n without the line j^{l^m}, then, all intersections $j^{l^m} \times l^n$ fit $j^{l^m} \times h^{l^n}$ and the line j^{l^m} fits all l^n.*

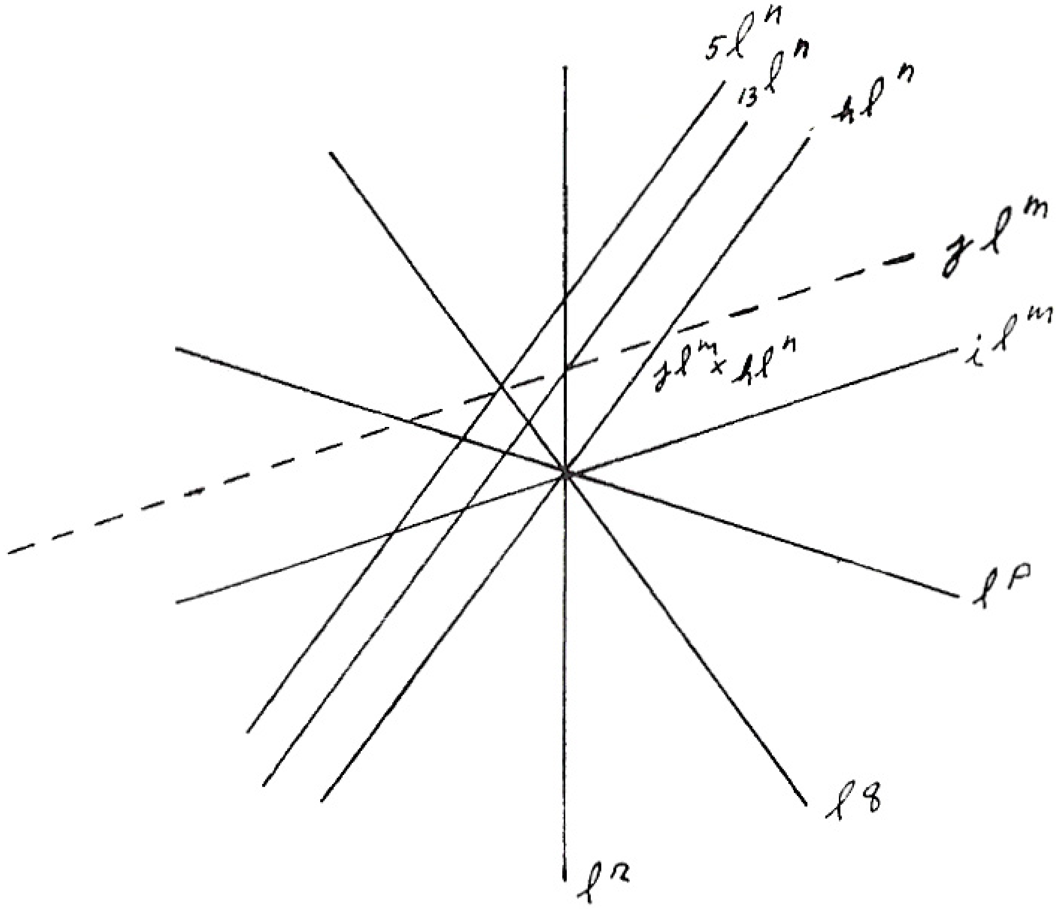


Figure 133

Lemma 12.1.5. *If j^{l^m} fits h^{l^n} then all l^p, $p \neq m, n \times j^{l^m}$ fit $jlm \times hln$ and if h^{l^n} fits all l^p without j^{l^m} them j^{l^m} fits all l^p.*

Theorem 12.1.6. *If a pattern fits and a line j^{l^m} is added and j^{l^m} fits some line j^{l^n}, $n \neq m$, then the new pattern including j^{l^m} fits.*

A pattern fits if all the intersections on all the lines fit. (Definition 5) To prove the new pattern fits we must prove all the new intersections fit the lines they are on.

1. *There are no new intersections on the lines k^{l^m}, $k \neq j$, therefore all the intersections on these lines fit.*

2. *All the intersections* $j^{l^m} \times l^n$ *fit* $j^{l^m} \times h^{l^m}$ *and the line* $j^{l^m} \times h^{l^m}$ *and the line* j^{l^m} *fits all* l^n*. (Lemma 4) Therefore all the intersections of the lines* l^n *fit.*

3. *All the intersections* $l^{p,p\neq m,n} \times j^{l^m}$ *fit* $j^{l^m} \times h^{l^n}$*. (Lemma 5) Therefore, the intersections of line* j^{l^m} *fit* $j^{l^m} \times h^{l^n}$*. Therefore, the intersections fit on all the lines* l^m *(step 1) and* j^{l^m} *fits all* $l^{p,p\neq m,n}$ *(Lemma 5) which means that all the intersections on all the lines* $l^{p,p\neq m,n}$ *fit and our theorem is thus proved.*

12.2 Three Dimensions

12.2.1 Definition 1

In three dimensions we have a pattern made up of the diameter lines through vertices and face midpoints of the icosahedron.

The lines through the vertices are the A lines.

The lines through the face midpoints lines are B lines.

There can be other lines parallel to the original 16 lines. They are given the name of the line they are parallel to.

We name a line

$$i^{A^{n^1}}$$

$$n = 1, 2, 3, 4, 5, 6$$
$$i = 1, 2, ...$$

$$j^{B^{m^1}}$$

$$m = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$$
$$j = 1, 2, ...$$

or generally

$$h^{X_i^{p^1}}$$

$$p = 1, 2 \ldots 10$$
$$h = 1, 2, \ldots$$
$$X = A, B$$

12.2.2 Definition 2

All lines of a 3D pattern are connected directly or through other intersections to the original pattern of 16 lines.

12.2.3 Definition 3

Intersections on A lines "fit" if all the intervals between them can be expressed as polynomials

$$s_1 AT^{r_1} + s_2 AT^{r_2} + \ldots\ s_n AT^{r_n} = f_i(AT) \tag{12.8}$$

$s_i, r_i =$ integers and $A =$ an interval of length.

12.2.4 Definition 4

Intersections on B lines "fit" if all intervals between them can be expressed as polynomials.

$$s_1 BT^{r_1} + s_2 BT^{r_2} + \ldots\ s_n BT^{r_n} = f_i(BT) \tag{12.9}$$

$s_i, r_i =$ integers and $B =$ an interval of length.

12.2.5 Definition 5

$$B = \frac{\cos(\frac{\pi}{6})}{\cos(\frac{\pi}{10})} A \tag{12.10}$$

12.2.6 Definition 6

A 3D pattern “fits” if all the intersections on all the lines fit.

12.2.7 Definition 7

An R section is a plane containing two A lines and two B lines. The angle between the two A lines is ▯; the angle between the two B lines is ▯; the angle between an A and a B line is

$$\frac{\pi - \theta - \phi}{2} \text{ or } \frac{\pi - \theta - \phi}{2} \tag{12.11}$$

12.2.8 Definition 8

A T plane is a plane perpendicular to an A line. A T plane is named by this A line

$$T^{A^n}$$

Lemma 12.2.1. *There are five R sections perpendicular to each T plane. Each A line is contained in five different R sections. Each B line is contained in three different R sections.*

Lemma 12.2.2. *The projection of the interval A along any A line onto a T plane*

$$= A\sin(\theta) \tag{12.12}$$

12.2.9 Definition 9

$$k = \sin(\theta) \;\; or \;\; = 0.A \tag{12.13}$$

1: What does 0.A resolve to?

(if the interval is on the A line perpendicular to the T plane).

Lemma 12.2.3. *The projection of the interval B along any B line onto a T plane*

$$= B\sin(\frac{\pi - \theta + \phi}{2}) = k \; or \;\; = B\sin(\frac{\pi - \theta - \phi}{2} = T^{-1_k}) \tag{12.14}$$

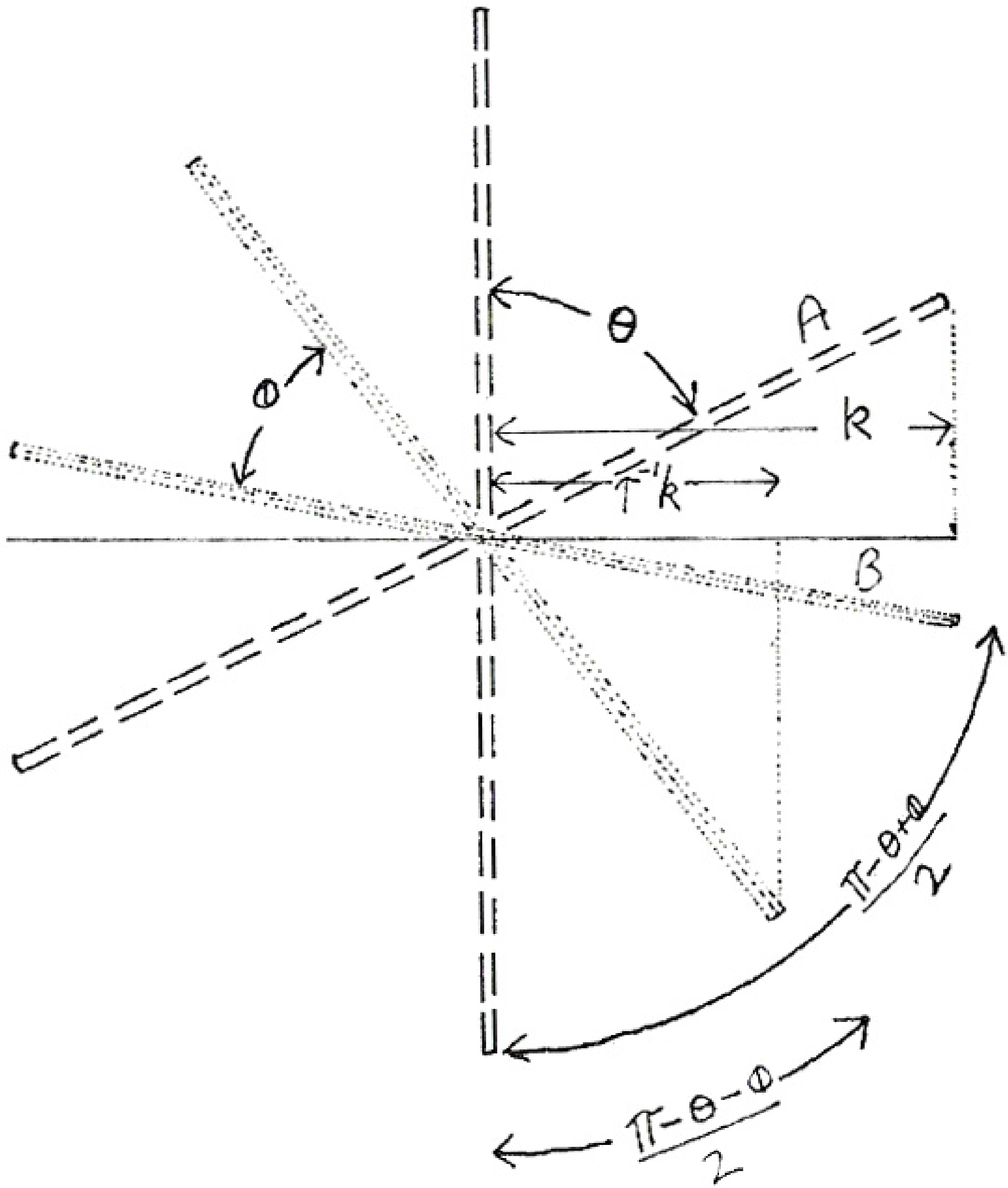


Figure 134

Lemma 12.2.4. *The projection onto any T plane of any interval along any line $k^{X_i^{r^1}}$ between any two intersections which fit in the 3D pattern is a polynomial*

$$s_1 kT^{r_1} + s_2 kT^{r_2} + ... + s_n kT^{r_n} = f_i(kT) \tag{12.15}$$

$$s_i, r_i = \textit{integers}$$
$$k = A\sin(\theta) = B\sin(\frac{\pi - \theta + \phi}{2})$$

Lemma 12.2.5. *The angles between the lines projected on a T plane are*

$$\frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5} \tag{12.16}$$

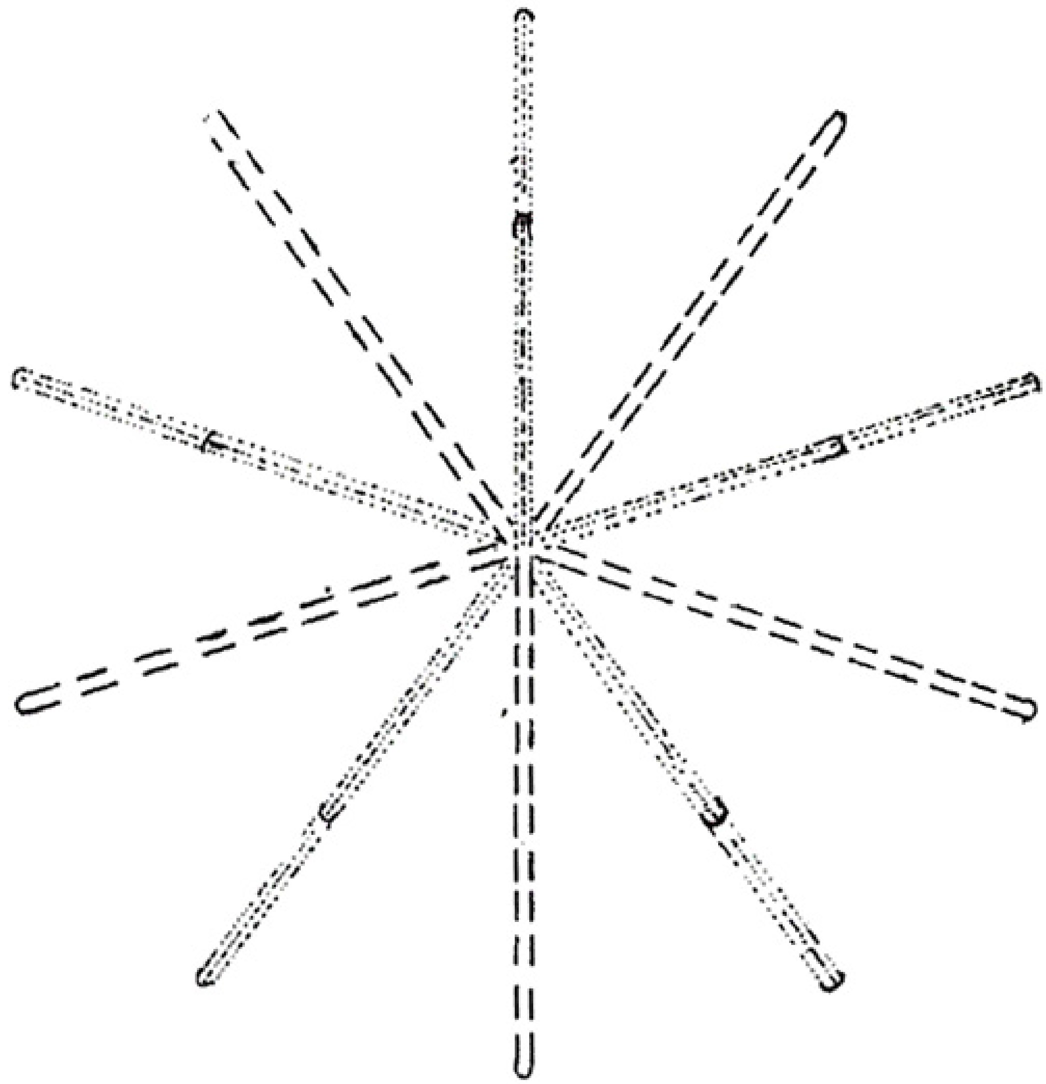


Figure 135

Lemma 12.2.6. *3D Patterns which fit project onto all T planes 2-dimensional patterns which fit.*

Lemma 12.2.7. *All intervals between intersections of lines of the 3D pattern appear as intervals between intersections in the projections on some T planes.*

For an intersection

$$i^{X_k^{n^l}} \times i^{X_j^{m^l}} \tag{12.17}$$

not to appear as an intersection in the pattern projected on a T plane, the intersecting lines must lie in one of the five perpendicular R sections.

At most, two different lines intersect $i^{X_k^{n^l}}$. The two intersections

$$i^{X_k^{n^l}} \text{ or } g^{X_t^{p^l}}$$

lie. Therefore, there is at least one T plane in which both 3D intersections project as 2-dimensional intersections.

Lemma 12.2.8. *If some pairs of intersections*

$$i^{X_k^{n^l}} \times h^{X_j^{m^l}} \text{ and } i^{X_k^{n^l}} \times g^{X_f^{p^l}} \tag{12.18}$$

did not fit in our 3D pattern, that is, we cannot find any polynomial $f_1(X_kT)$ that equals the interval between them, then also for some T plane we cannot find a polynomial $f_i(kT)$ which equals the interval between their projected intersections.

Lemma 12.2.9. *Any misfits between intersections of the 3D pattern project as misfits between intersections on some T plane. Therefore, a 3D pattern which projects on all T planes as a 2D Pattern which fits is a 3D pattern which fits.*

Theorem 12.2.10. *If a 3D pattern fits and a new line is added $k^{X_i^{n^l}}$ which intersects and fits some other line $h^{X_j^{n^l}}$ then the new 3D pattern including $k^{X_i^{n^l}}$ fits.*

Proof. In each T plane the projection of the line $k^{X_i^{n^l}}$ fits the projection of the line $h^{X_j^{m^l}}$. In each T plane the projection of the 3D pattern without $k^{X_i^{n^l}}$ fits. Therefore, the projection is with the line $k^{X_i^{n^l}}$ fits on each T plane and the theorem is proved. □

The coherence proof demonstrates that if one builds a structure using the A and B lines of the 31 zone star (the C lines may be used only within the forms defined by the A and B lines) and always follows the rule that new parts are added at intersections of existing parts or at points along existing parts which can be reached by subdividing a large part into component small parts, then no matter how far or intricately one builds, two extensions of two entirely different limbs of the same structure can always be locked back together in a perfect fit with a combination of our simple parts.

We have shown that the intervals between intersections are all equal to certain polynomials in T. Because

$$T^n = T^{n-1} + T^{n-2} \tag{12.19}$$

all the terms of any such polynomial can descend by subdivisions into a polynomial with only two terms-

$$f_i(AT) = rAT^n + sAT^{n-1} \tag{12.20}$$

There are many interesting side-lights to these investigations. One of which is that it is impossible to divide any one of our building blocks AT^n into equal pieces.

There are much shorter proofs of the coherence of this system, but the short proofs don't lead one through so many characteristics of the structure.

13 Joints

We have associated the thirty-one zone star throughout with the icosahedron and the dodecahedron. It also fits perfectly with the three smaller regular polyhedra. The tetrahedron, the cube and the octahedron fit inside the icosahedron and the dodecahedron. Their vertices touch a vertex, an edge midpoint or a face midpoint of the larger figure. This regular match between large and small figure positions the smaller figure so that regular patterns on the large figure project inwards as regular patterns on the small figure. In each case, either five or ten small figures fit at once within the larger figure.[1]

Each of the regular polyhedra is thus a convenient core from which to define the regular thirty-one zone star. The geometric regularities insure simplicity in the connections. Any one of the regular polyhedra can be used with the same pattern of flanges or holes on each of its faces as a connector for the thirty-one zone structural system.[2]

1 See illustrations in Cundy and Rollet's *Mathematical models* [CR61].

2 Note! In the case of the thirty-one zone pattern projected on the octahedron, four of the faces have a left-handed pattern. The drawing and the photo are of patterns with different handedness.

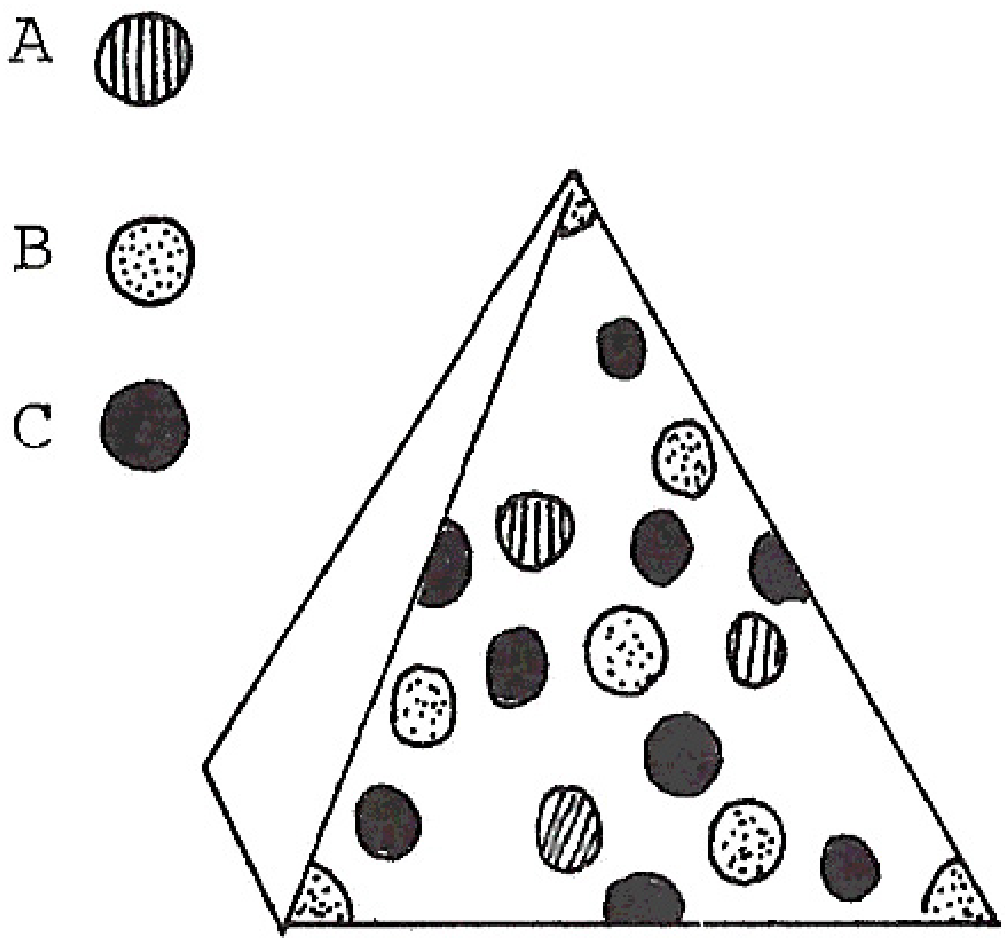


Figure 136: Tetrahedron

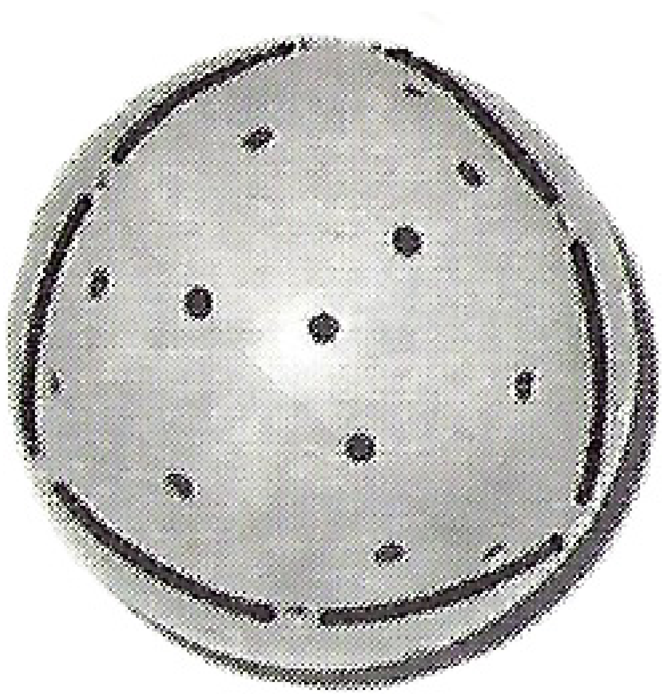


Figure 137: Tetrahedron Vertices

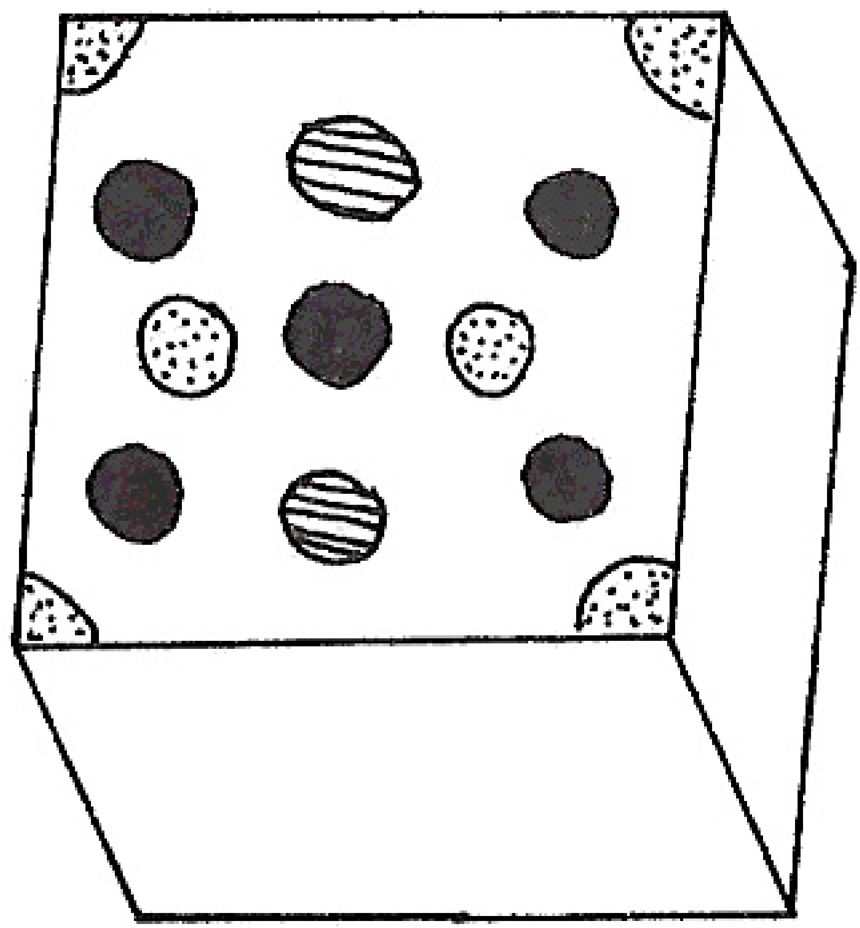


Figure 138: Cube

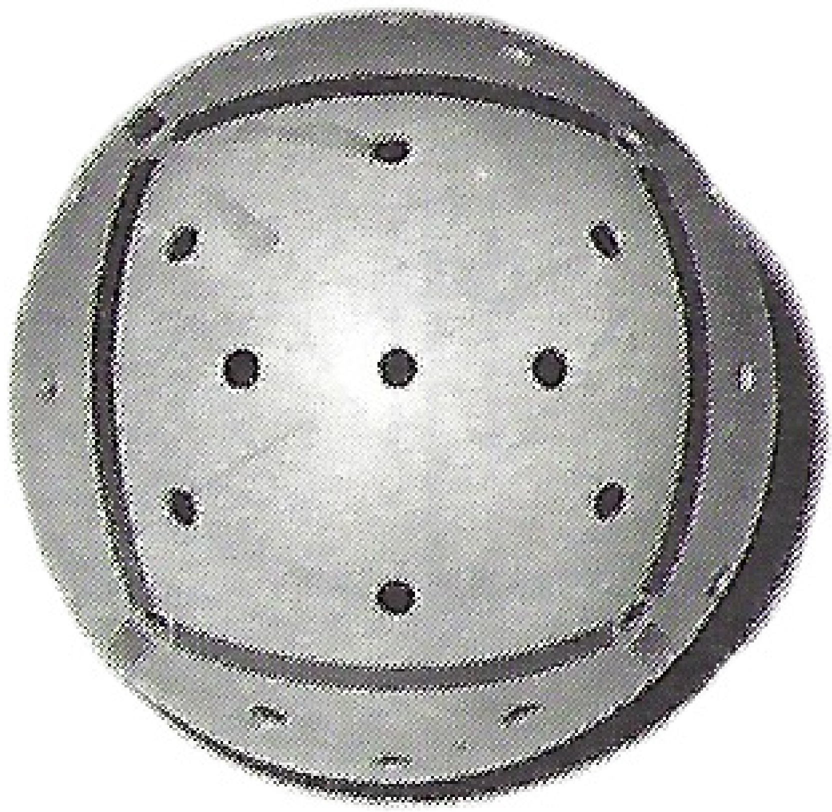


Figure 139: Cube Vertices

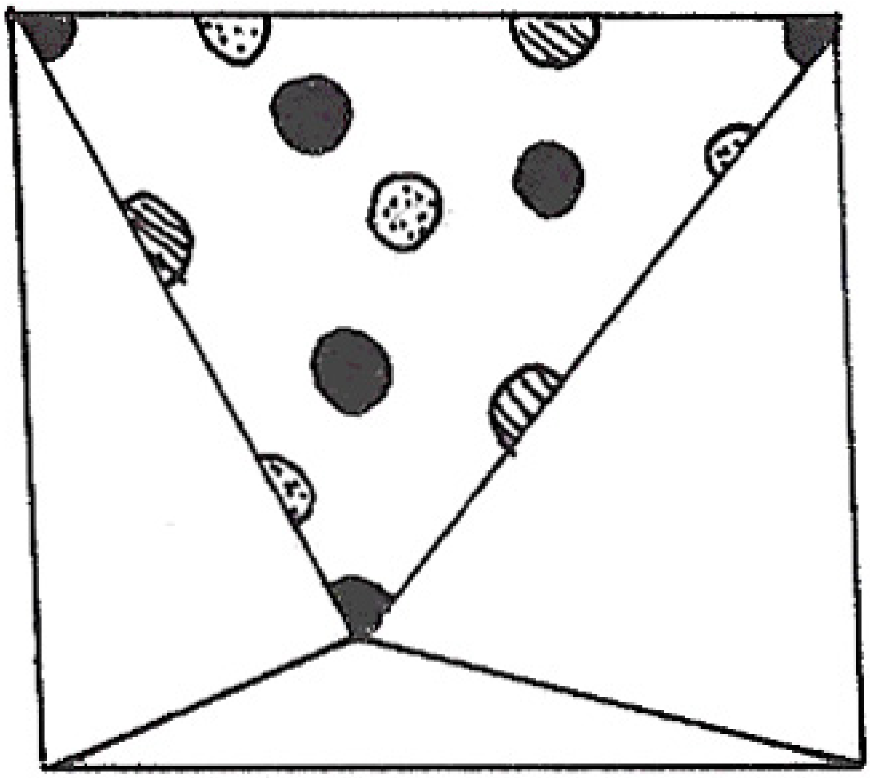


Figure 140: Octahedron

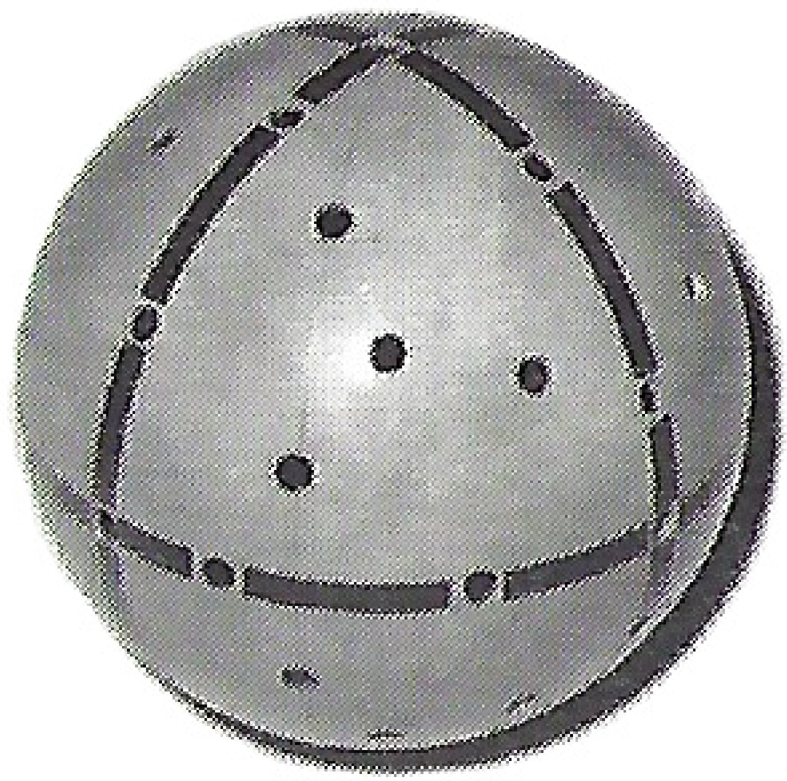


Figure 141: Octahedron Vertices

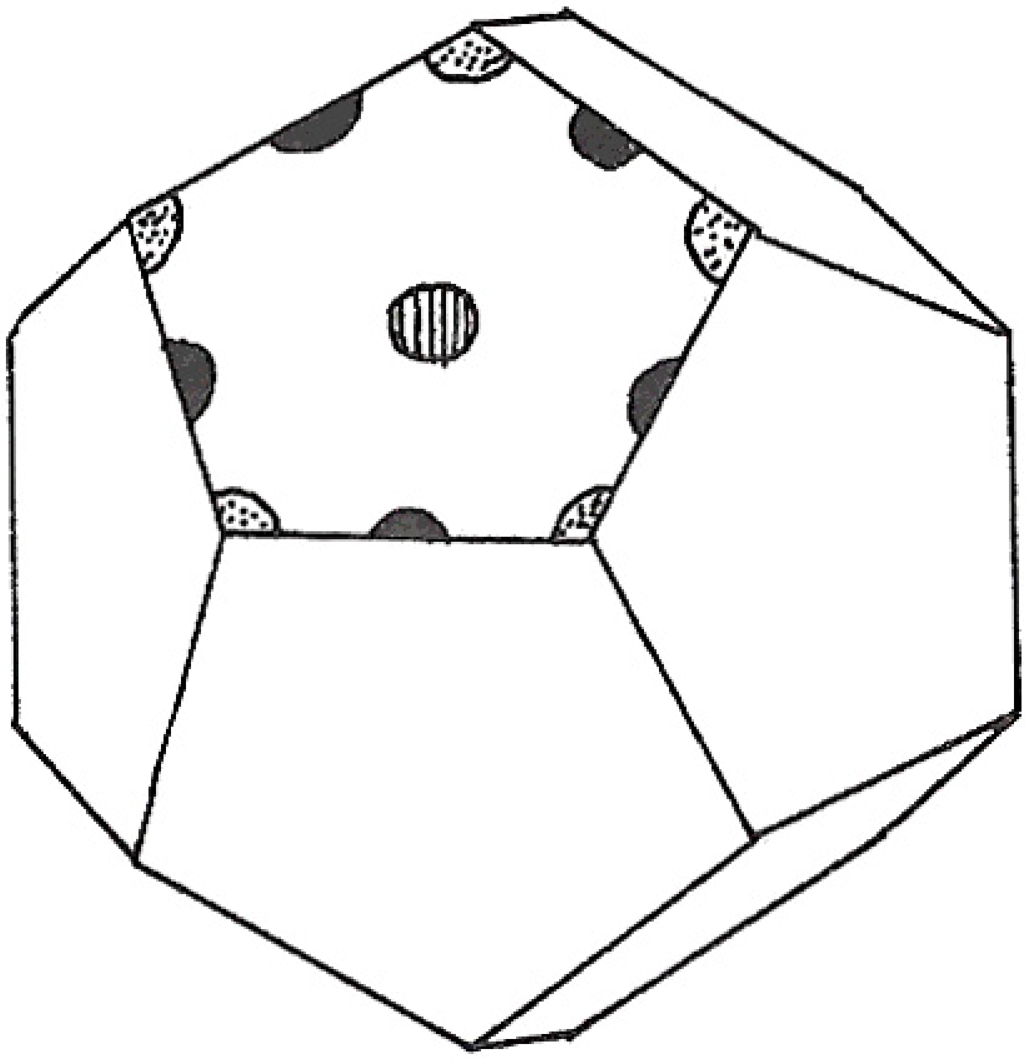


Figure 142: Dodecahedron

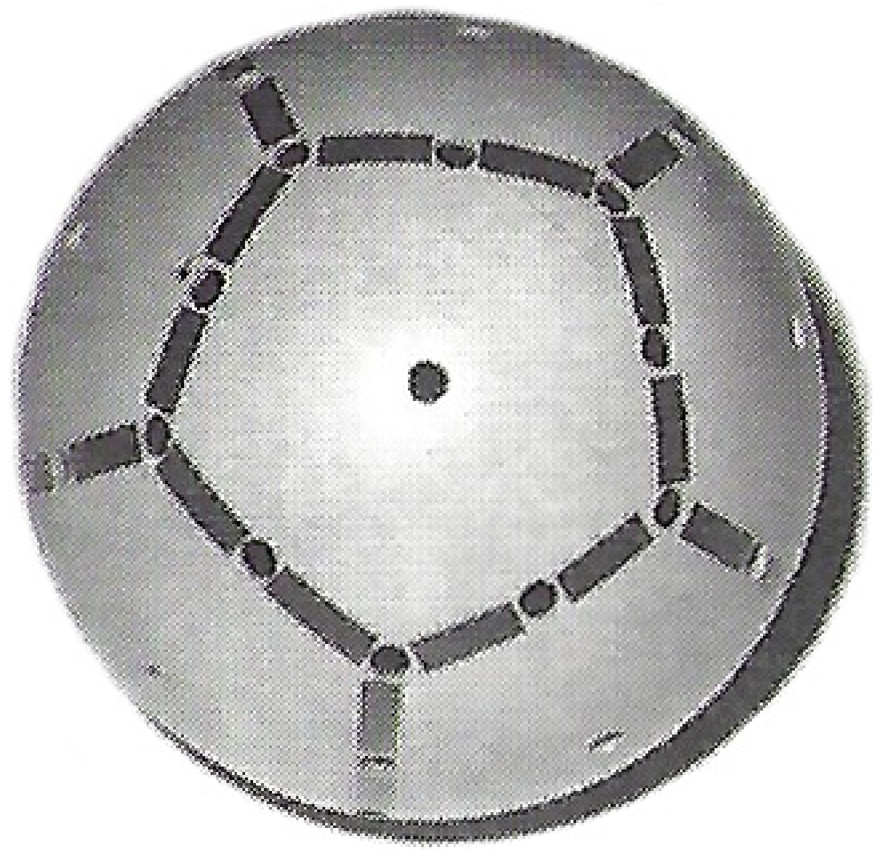


Figure 143: Dodecahedron Vertices

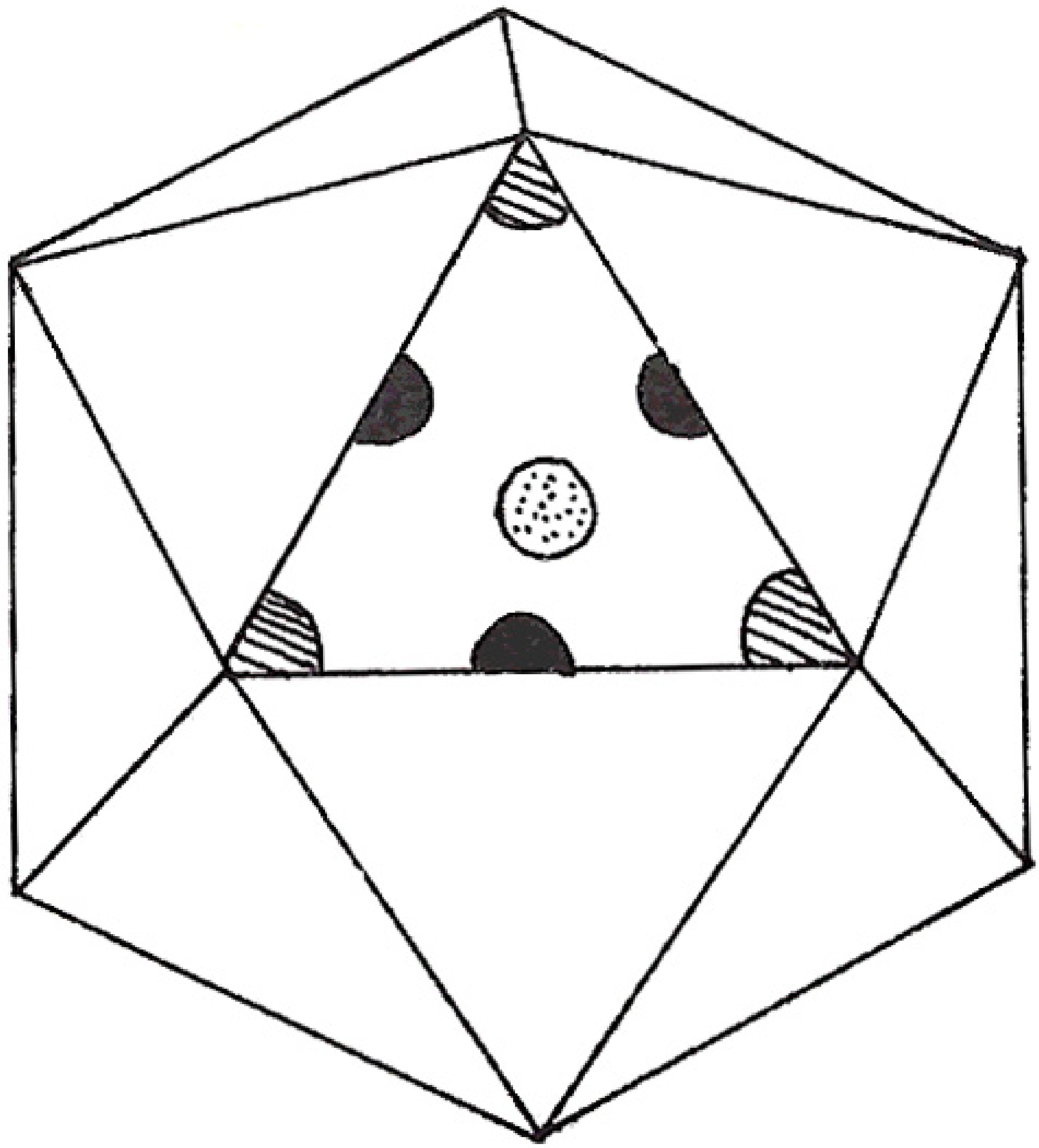


Figure 144: Icosahedron

Figure 145: Icosahedron Vertices

14 The Stars of Other Systems

In the design of a connector simplicity in one part may demand specialization and the establishment of hierarchies among other parts.

The sketches of a joint of an icosahedron show examples of different ways of joining the five edges of an icosahedron. The parts remain identical if they trade or share with each other at the joint.

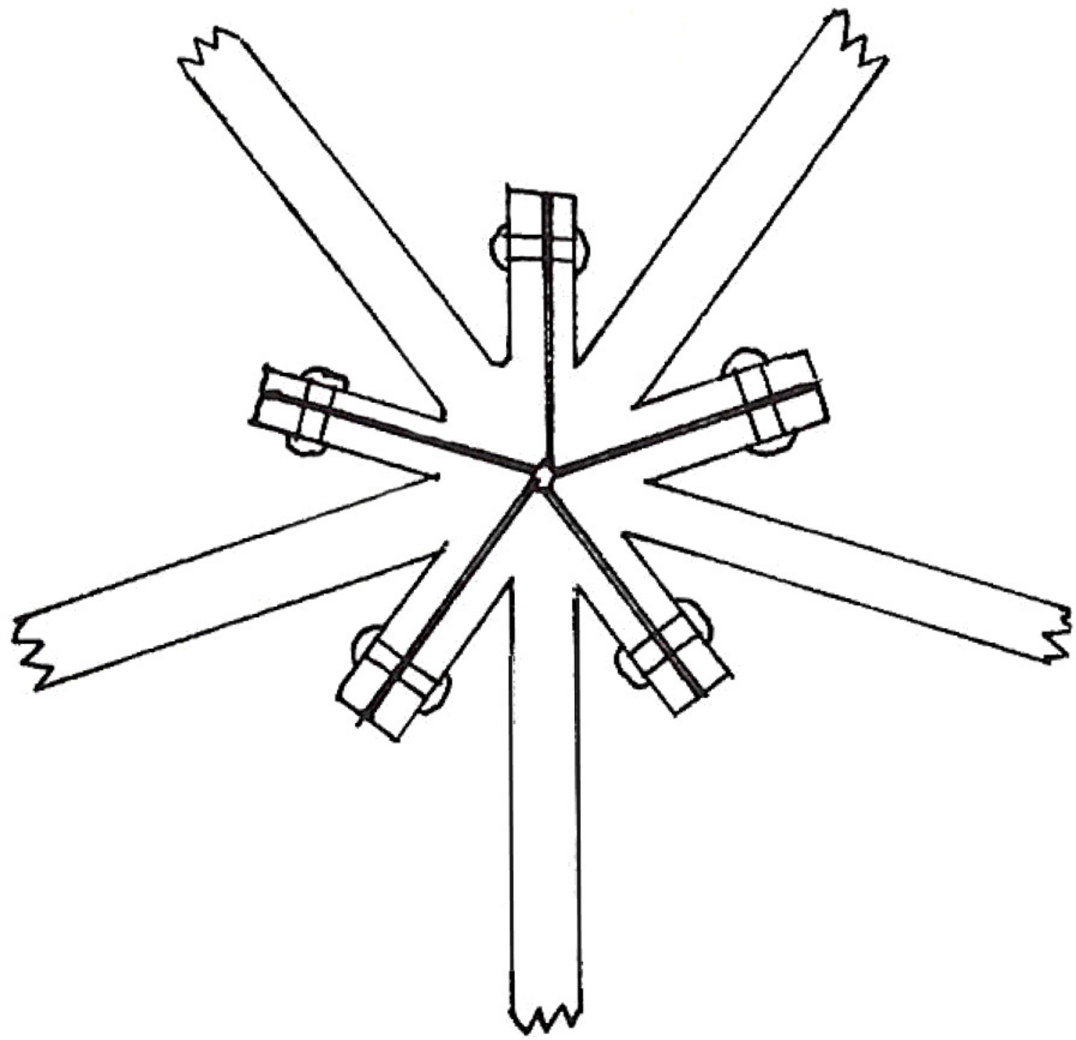


Figure 146: Sharing

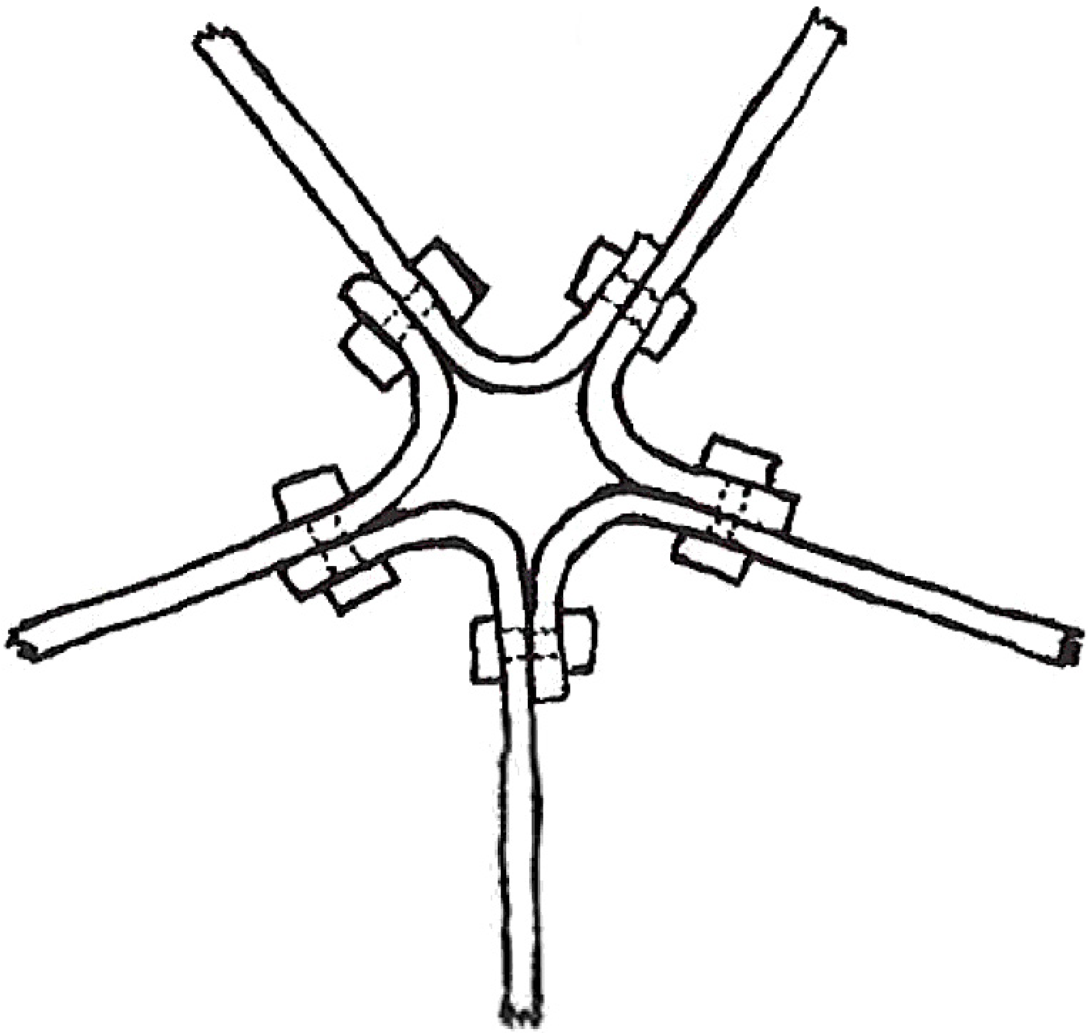


Figure 147: Trading

When some parts dominate others, there is a hierarchy created and the parts must be differentiated. In our illustration, we show five edges of an icosahedron stacked on top of one another. With the icosahedron, it is possible to have both ends of each edge occupy similar positions, but this is not always true.

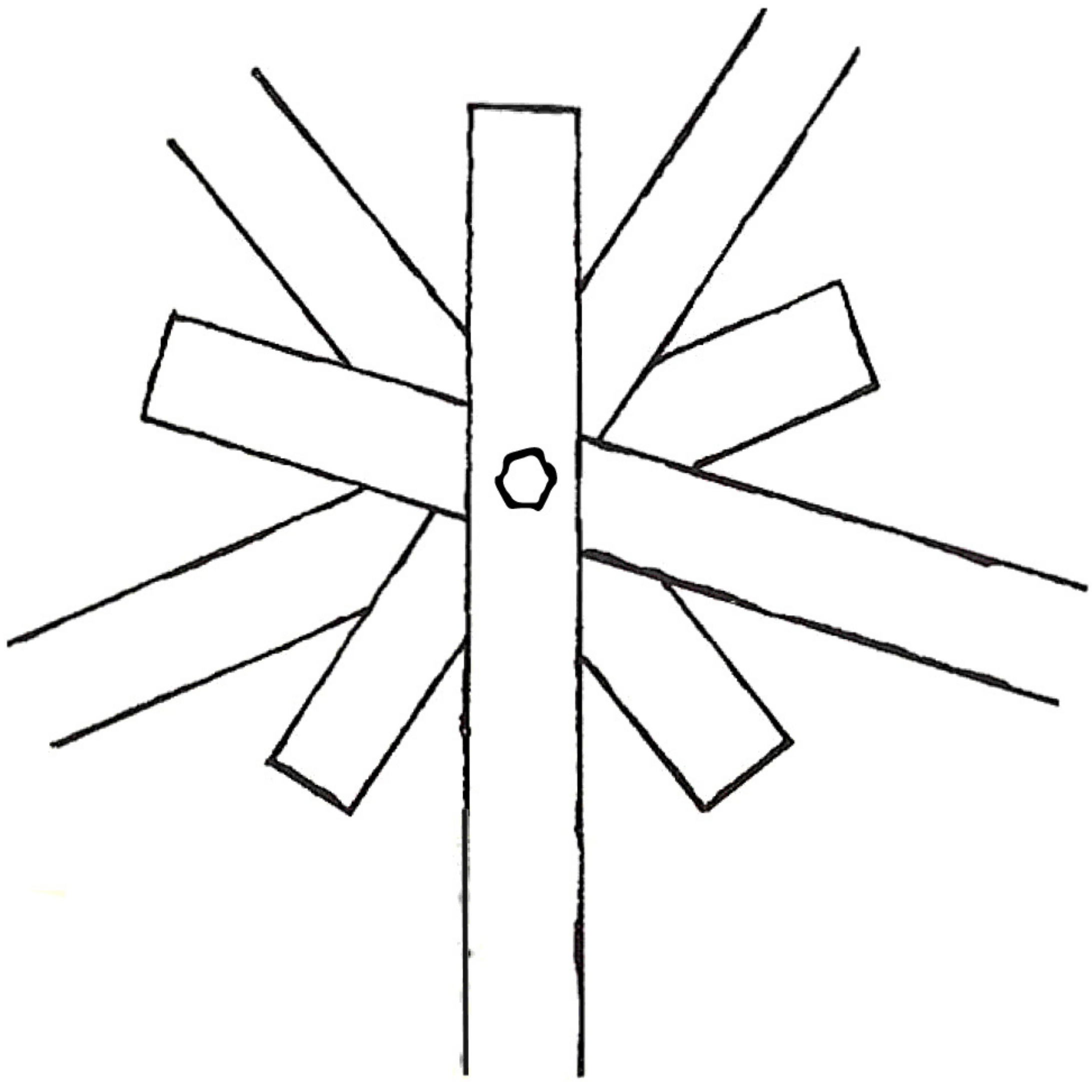


Figure 148: Dominating

The creation of one hierarchy may create the need for more. For instance, if you wish to use a hierarchy of stacked edges to make a simple triangle, then you must also create a hierarchy between ends of the same edge. The two ends of the same edge have different ranks.

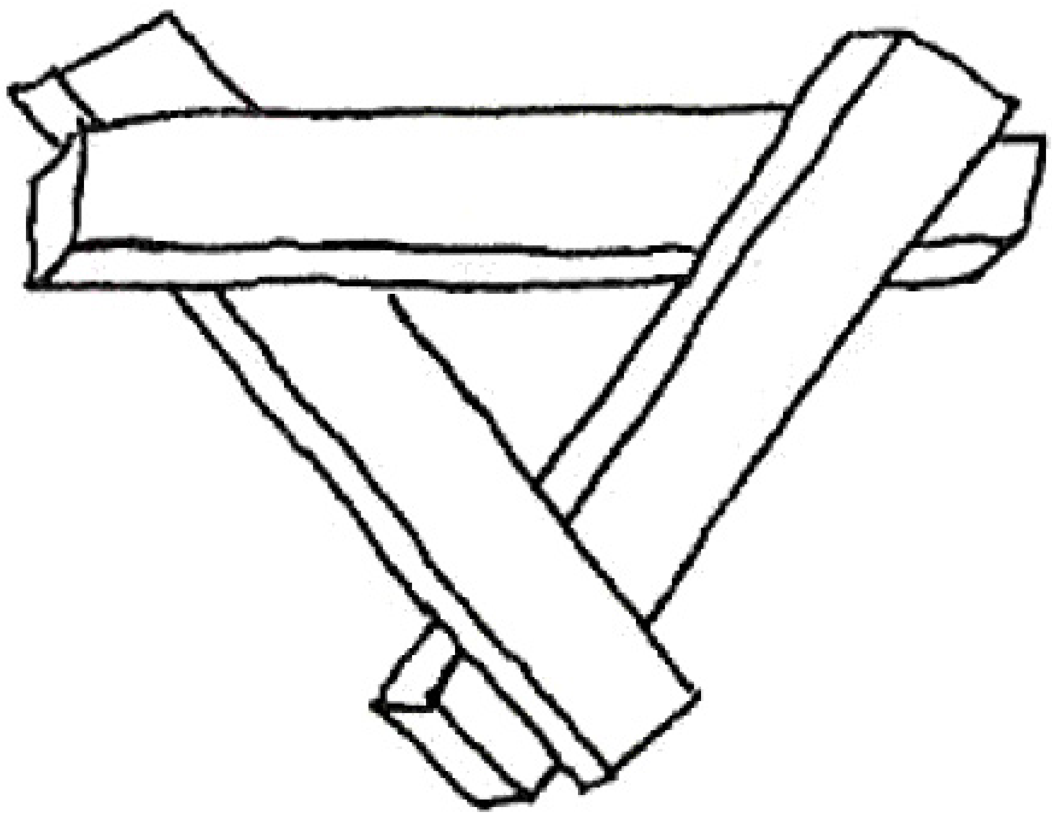


Figure 149: Stacked Edges of a Triangle

Very often we don't have the structural members actually touching each other. Then we avoid the edges' problem of sharing, trading or dominating—we push the entire problem on to the joint. The joint must accommodate all the connecting members, and it shouldn't demand that they have elaborate ends or different kinds of ends. The joint must also be strong and inexpensive.

If the joint is a ball and the A, B and C connections are simply holes which the members screw into. All holes of the same type are identical, and the ends of all structural members are identical.

This is a wonderful feature because you can't make mistakes—there are no right and wrong holes of a certain type. The joint shown in Figure 160 has 12 threaded holes for the A lines in the thirty-one zone system. Joints made this way are expensive, and the connectors at the end of the structural members are expensive. A common inexpensive connecting system is a joint made of multiple interconnecting flanges with the flattened ends of the structural members bolting against the flange of the joint. Joints made of flanges can be simple flat stampings that lock together or are welded together.

Flanges bring problems. What if you must connect two joints, and you find that once you bolt one end of your member to one joint it can't bolt to the other because the flange is turned the wrong way? The specialization of a flat end and a flange has brought problems. The end of the structural member could be made to swivel. That is expensive. The joint could be made so that this problem never arises. Unfortunately, our second solution is not always possible. To make a thirty-one zone joint where each connection is a flat flange and which is impossible to put together incorrectly, you have to devise a pattern that passes through each connecting point only once and is regular (you can't tell the pattern that surrounds a connection from the pattern that surrounds any other connection of the same type).

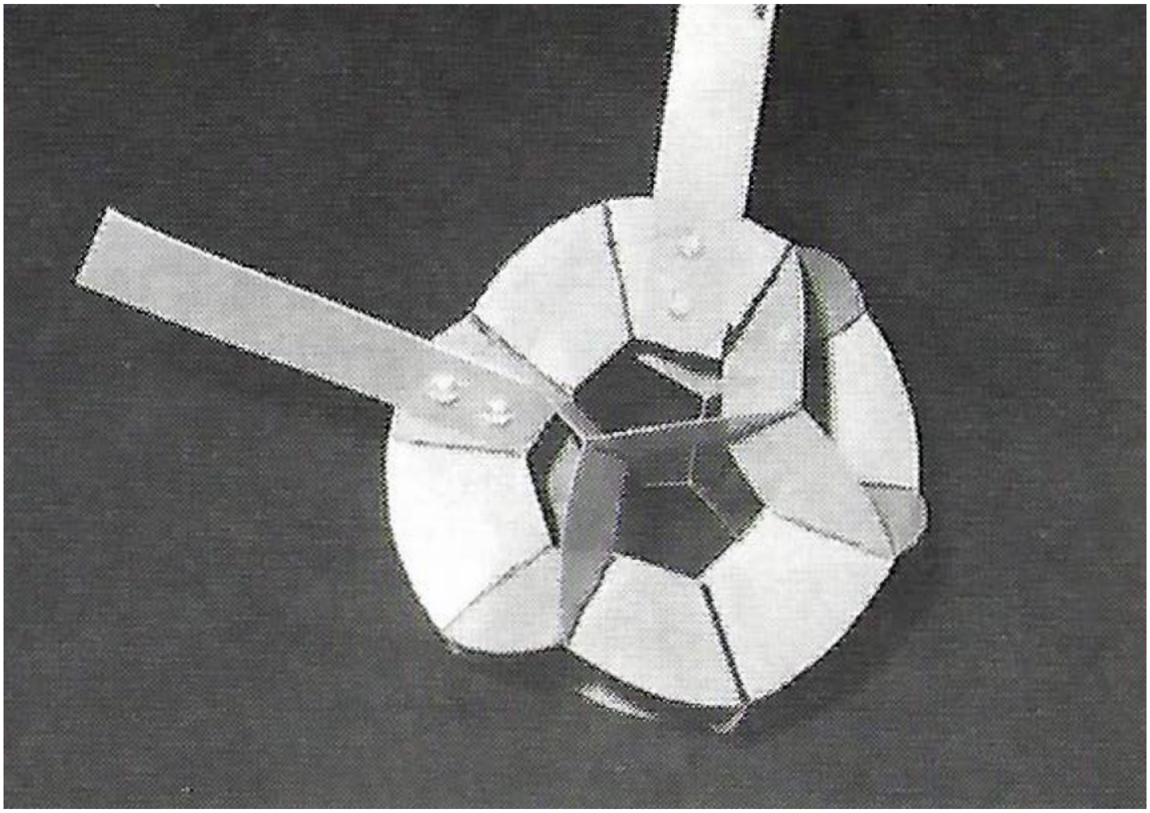


Figure 150: Flanges of an icosahedron or dodecahedron

This is impossible to do in the case of the thirty-one zone star. You can make a flange joint for the thirty C connections. The C lines can have the specialization of flange orientation while still avoiding the clumsiness of having to arrange themselves in a hierarchy. The flanges are the edges of an icosahedron or a dodecahedron.

Three interlocking rectangles at right angles to each other form a flange joint for the 12 A lines, 6 C lines and 12 B lines.

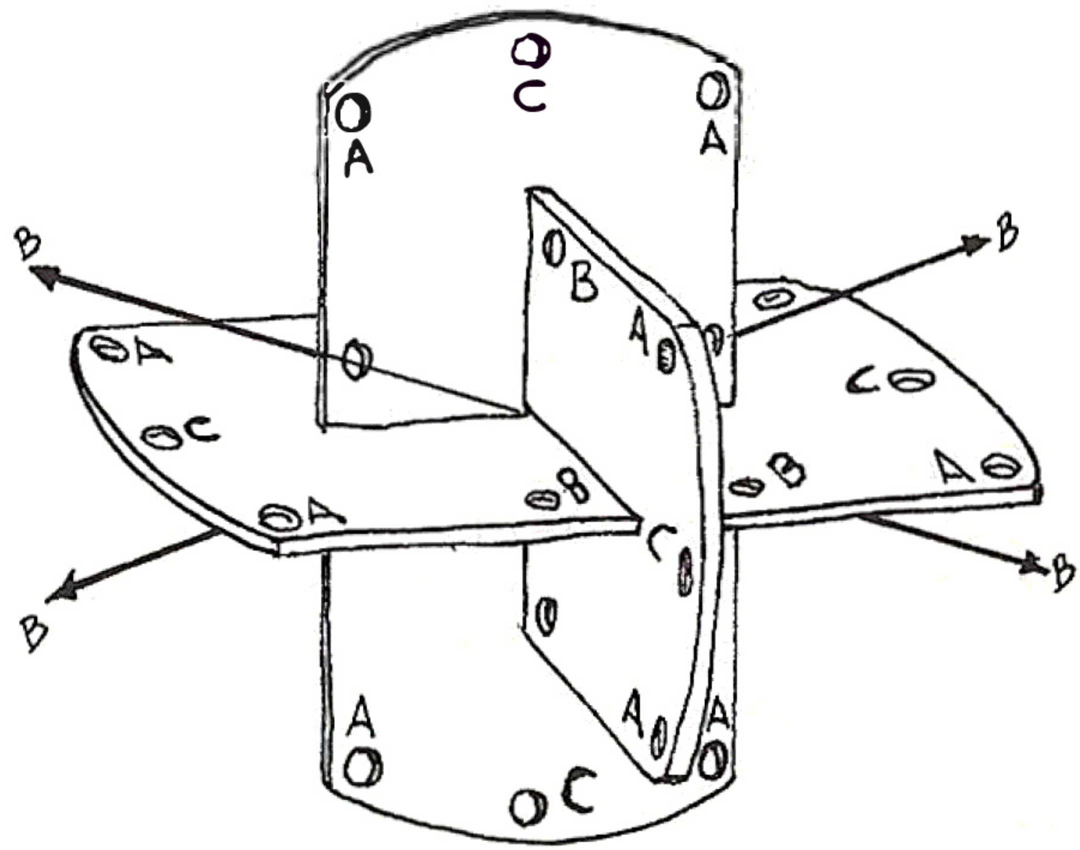


Figure 151: Interlocking rectangles forming a flange joint

Note! These are three perpendicular R sections.

There is a mistake proof flange joint for both A and C connections if one hierarchy is introduced. You must always orient the joint to suit the A lines.

This joint is shown in the photo of the wooden model of the three intersecting planes.

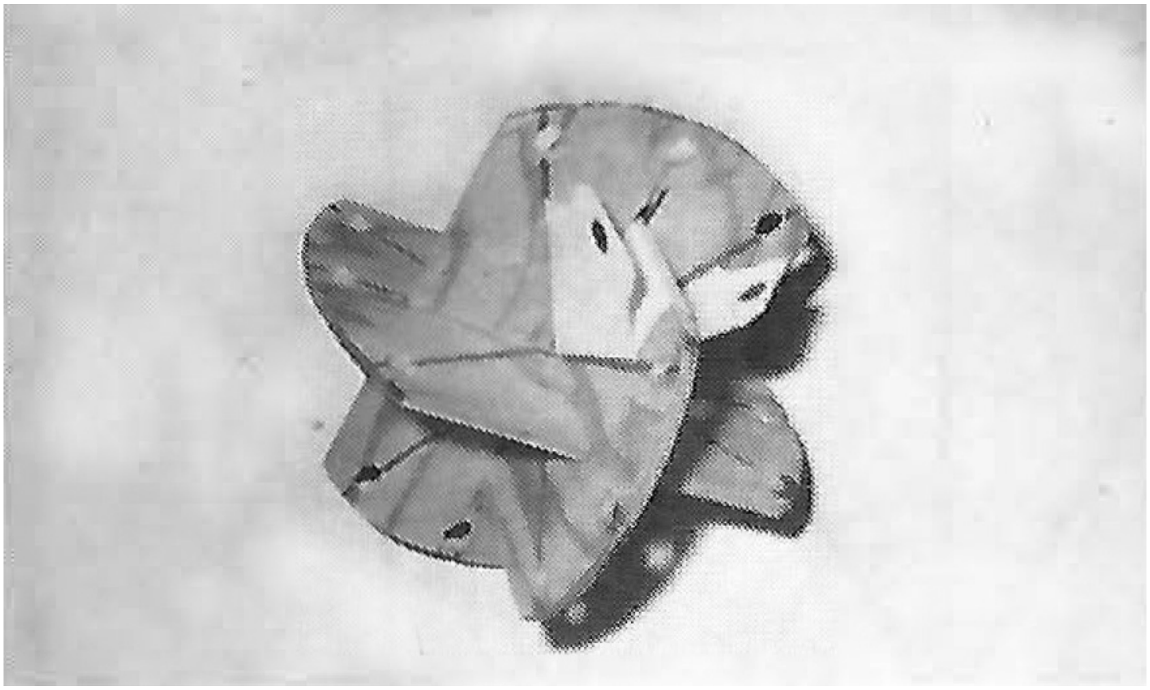


Figure 152: Model of intersecting planes

Two of the flanges for the missing C lines are added. This joint is completely regular until the addition of these C flanges—they are not in the same plane as their neighboring A flanges. We thus have two types and lose our regularity.

A good flange joint for the thirty-one zone star is made of six interlocking disks—five R sections and one T section.

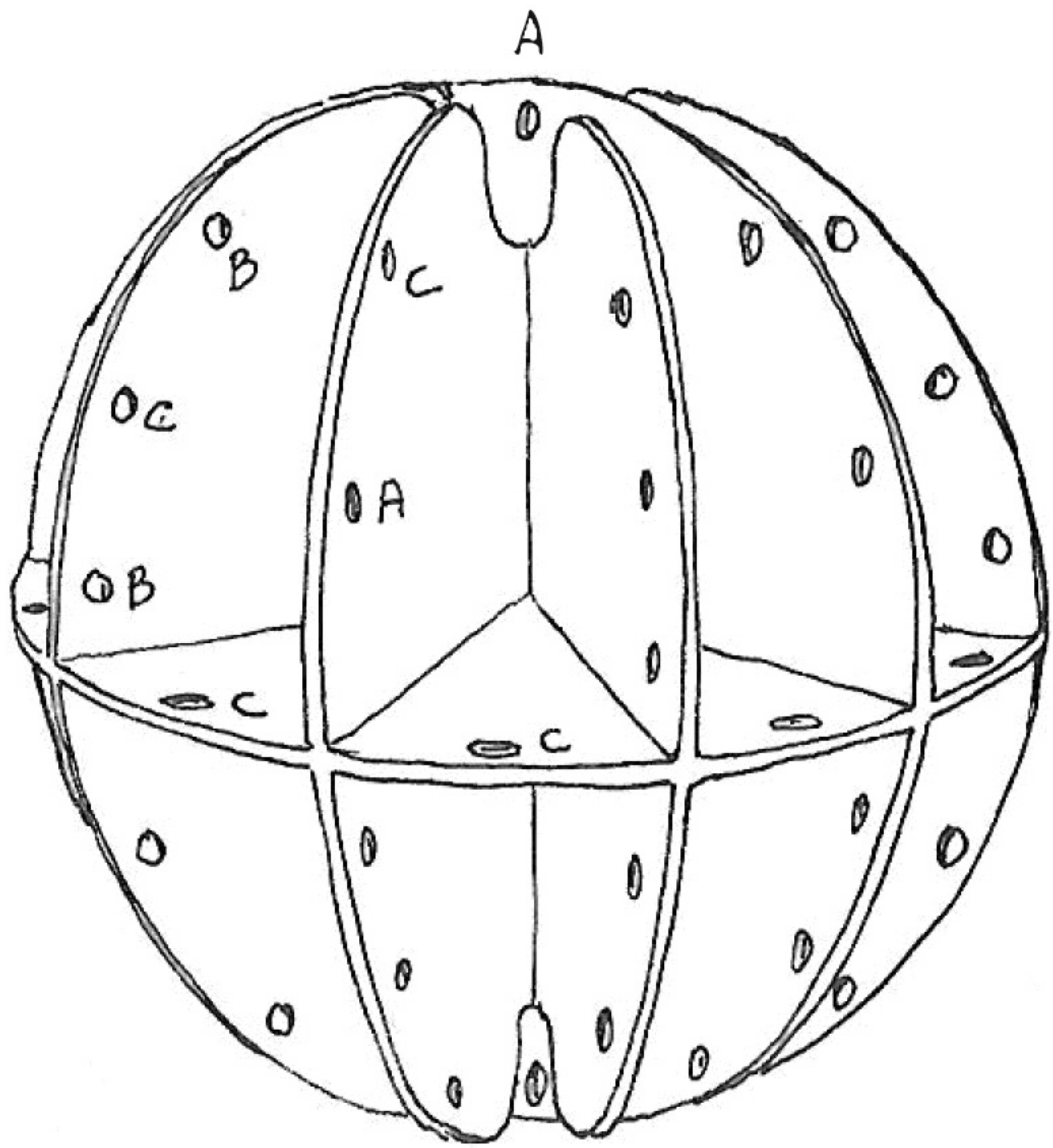


Figure 153: Six interlocking disks

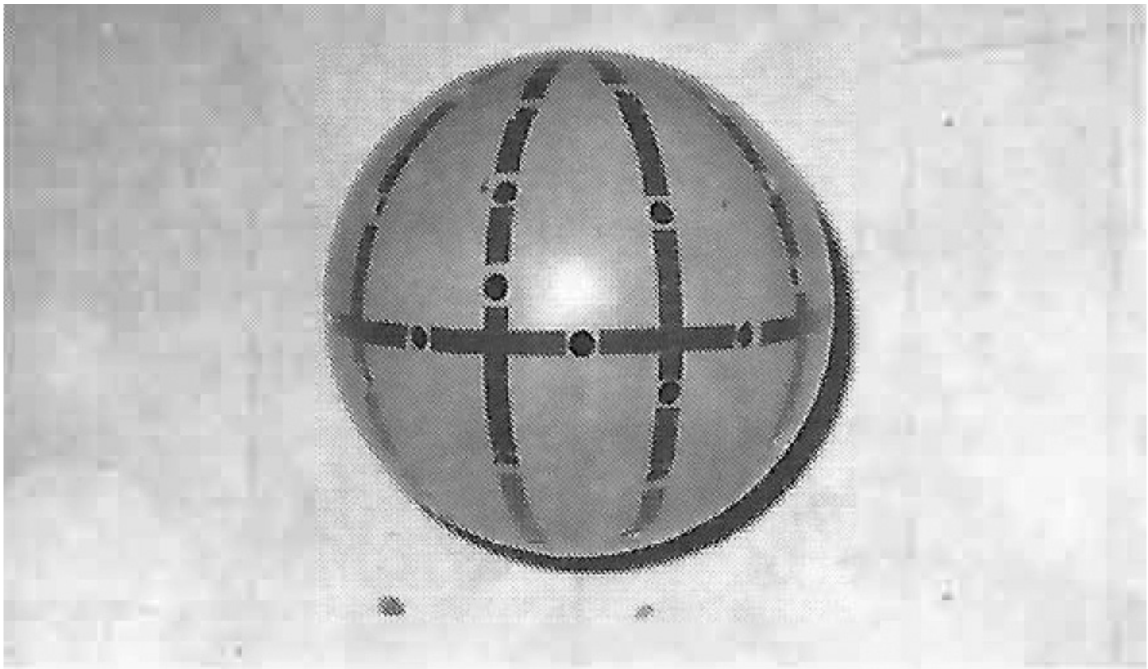


Figure 154: Vertices on a sphere

This creates an irregular icosahedron which passes through each of the sixty-two connecting points and decides automatically the orientation of sixty of the flanges. The pole points of this figure are A connections and one of the five intersecting R sections must dominate the others to determine the flange orientation. This joint can have part or all of the R sections in place depending on which connections are needed.

14.1 Octet Truss

The widely used octet truss is based on the star that passes through the midpoints of the edges of a cube. (Or, equivalently, the midpoints of the edges of an octahedron, the midpoints of the faces of a rhombic dodecahedron, the vertices of a cuboctahedron.)

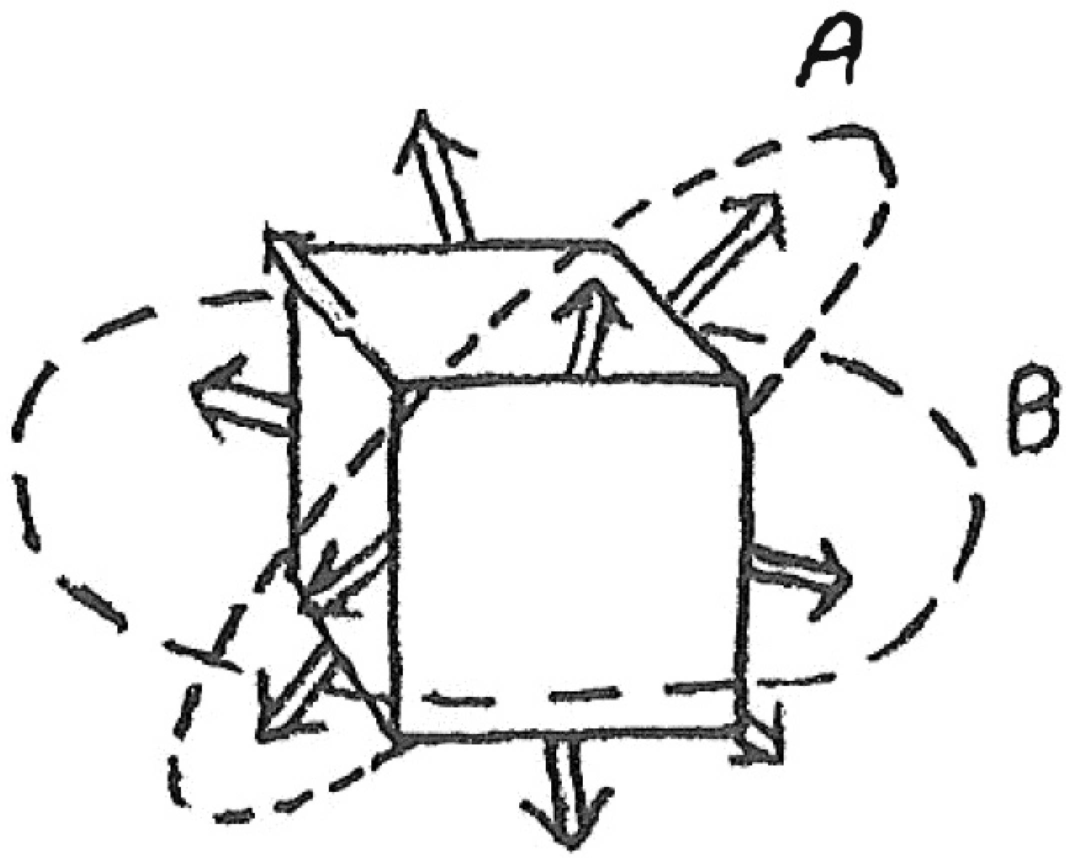


Figure 155: Star for octet truss

This is a singular star with four sections (A sections in drawing) containing three lines at 60° to each other and three sections with two lines at right angles (B sections). The zonohedron whose edges are parallel to the lines of the star is the truncated octahedron.

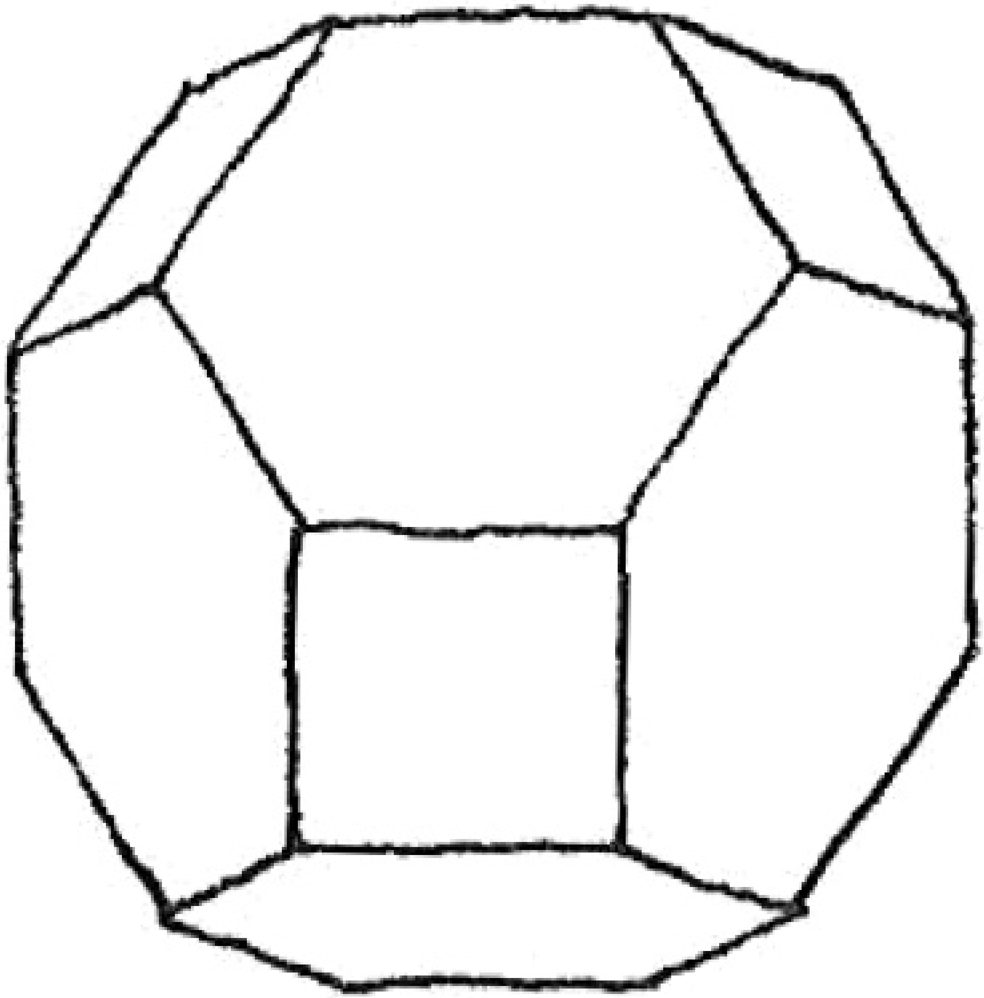


Figure 156: Truncated Octahedron

14.2 The Mero Space Grid System

This system is based on the octet truss star plus three more zones which pass through the face midpoints of the cube which has the other lines passing through its edge midpoints. This singular star has three sections (B sections) with four lines intersecting at 45° to each other, four sections (A sections) with three lines at 60° to each other and six sections (C sections) with two lines at 90° to each other.

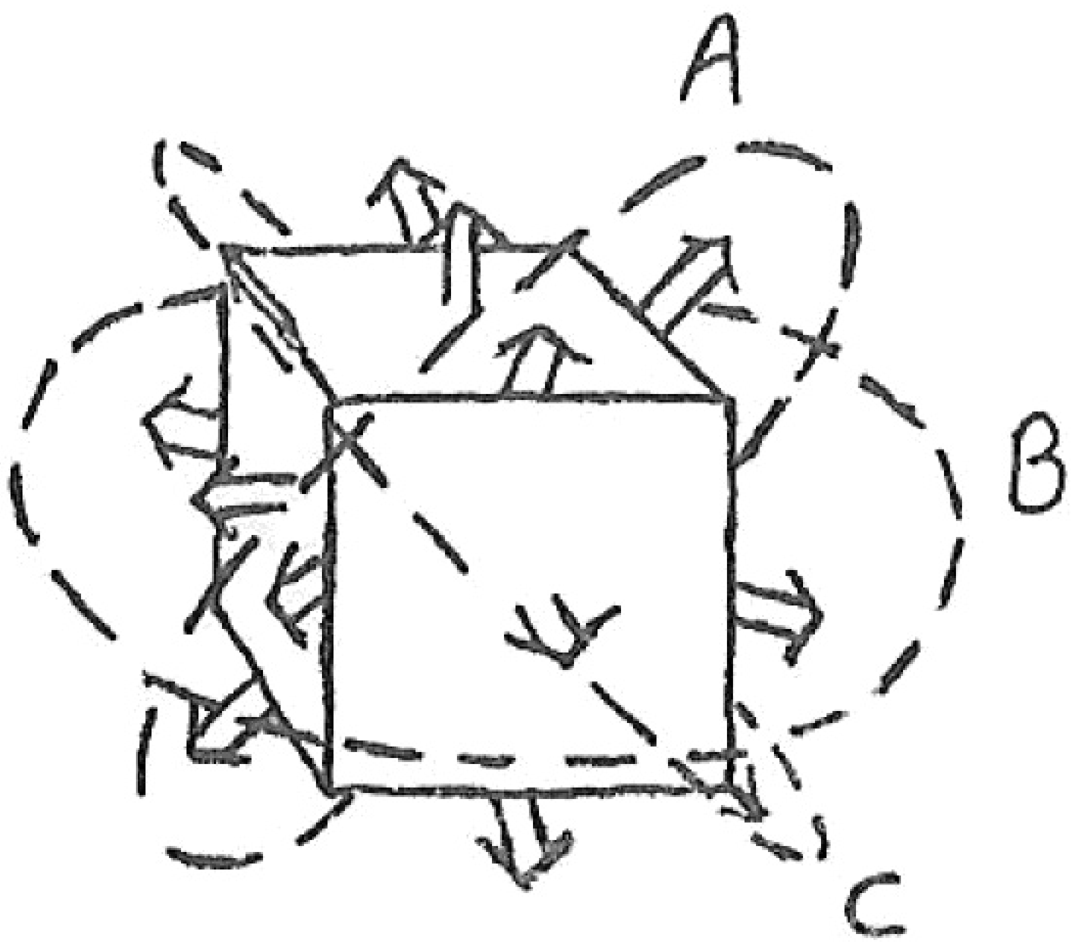


Figure 157: Star for MERO system

The zonohedron associated with this star is the truncated cuboctahedron.

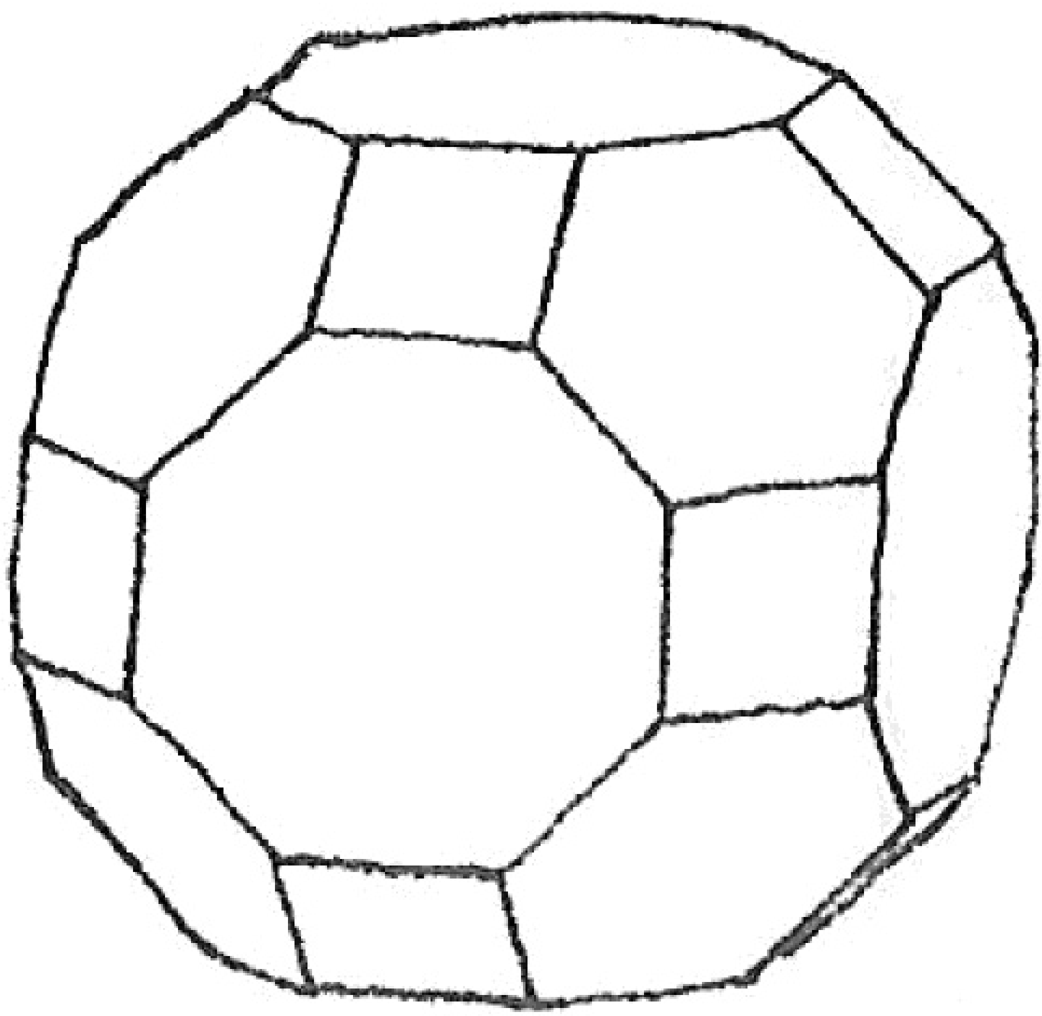
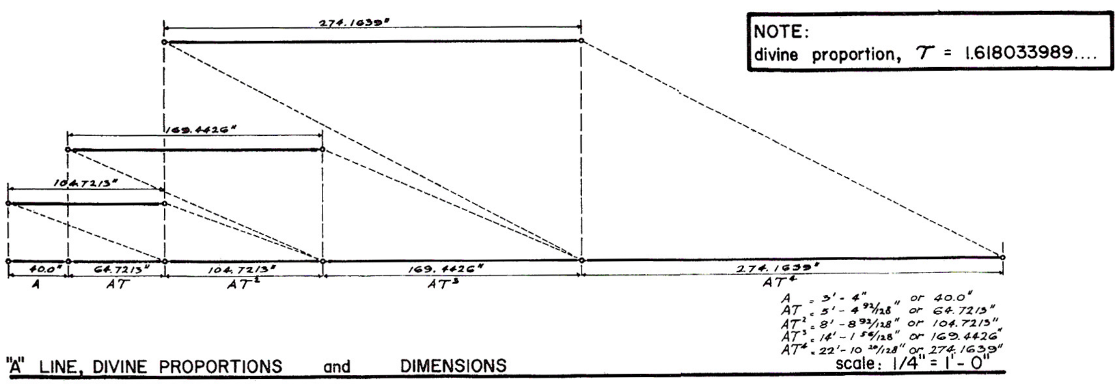
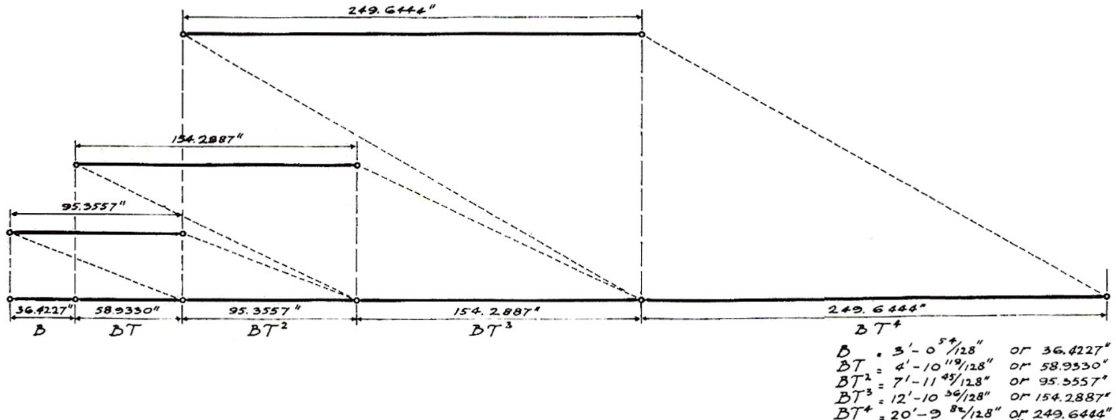


Figure 158: Truncated Cubooctahedron

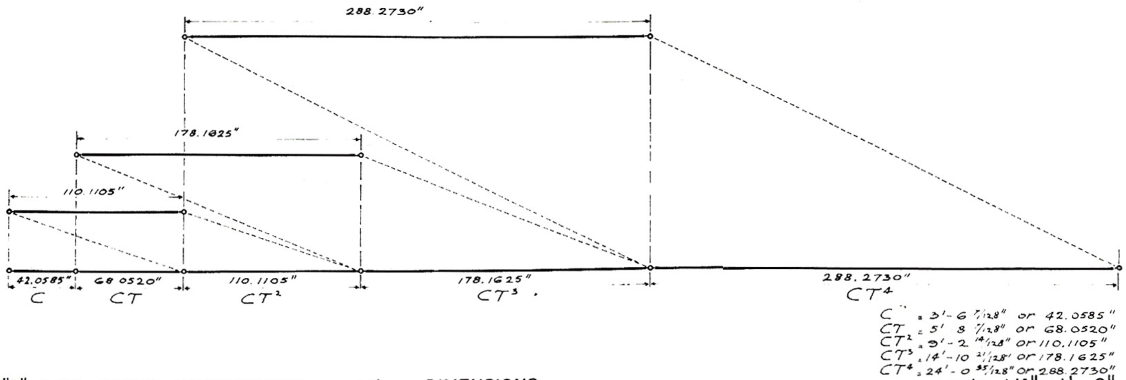
15 Hardware


NOTE:
divine proportion, τ = 1.618033989....
274.1639"
169.4426"
104.7213"
40.0"
64.7213"
104.7213"
169.4426"
274.1639"
A
AT
AT²
AT³
AT⁴
A = 3'-4" or 40.0"
AT = 5'-4 92/128" or 64.7213"
AT² = 8'-8 92/128" or 104.7213"
AT³ = 14'-1 56/128" or 169.4426"
AT⁴ = 22'-10 20/128" or 274.1639"
"A" LINE, DIVINE PROPORTIONS and DIMENSIONS
scale: 1/4" = 1'-0"

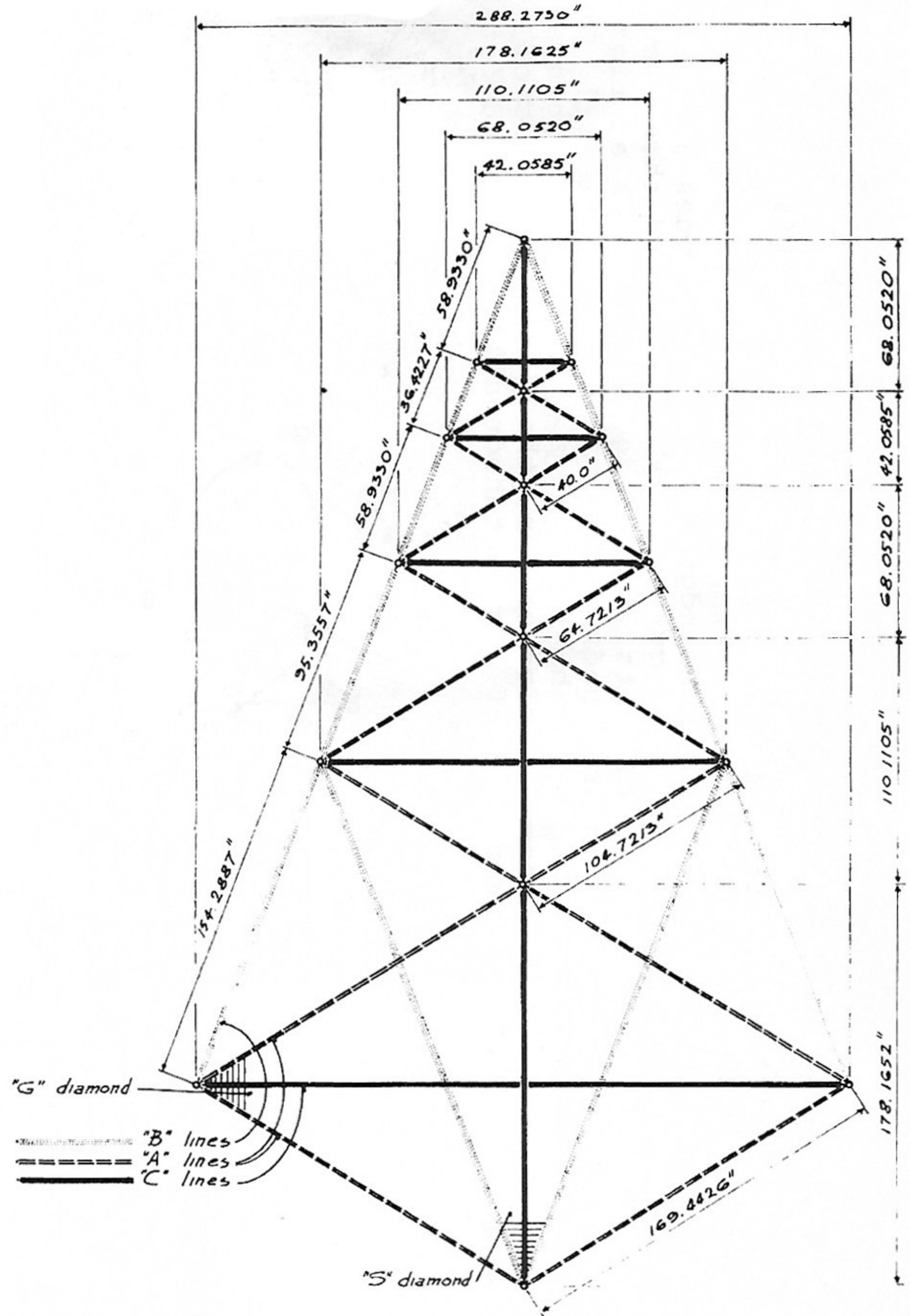


"B" LINE, DIVINE PROPORTIONS and DIMENSIONS

scale: 1/4" = 1'-0"



"C" LINE, DIVINE PROPORTIONS and DIMENSIONS scale: 1/4" = 1' - 0"



TYPICAL FRAMING PLAN scale: 1/4" = 1' - 0"

Lengths of A, B, and C lines.

Structural members where $A = 40''$.

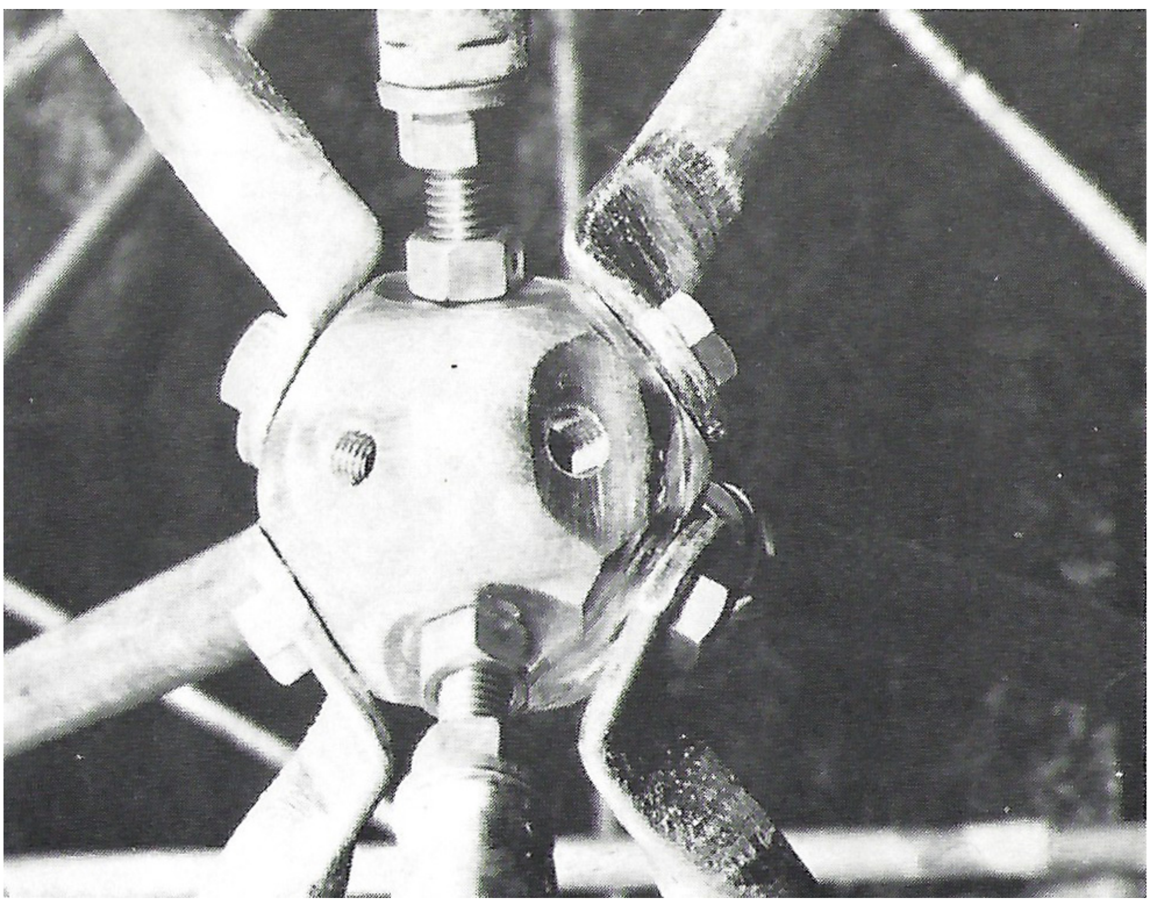


Figure 160: Six Zone Joint

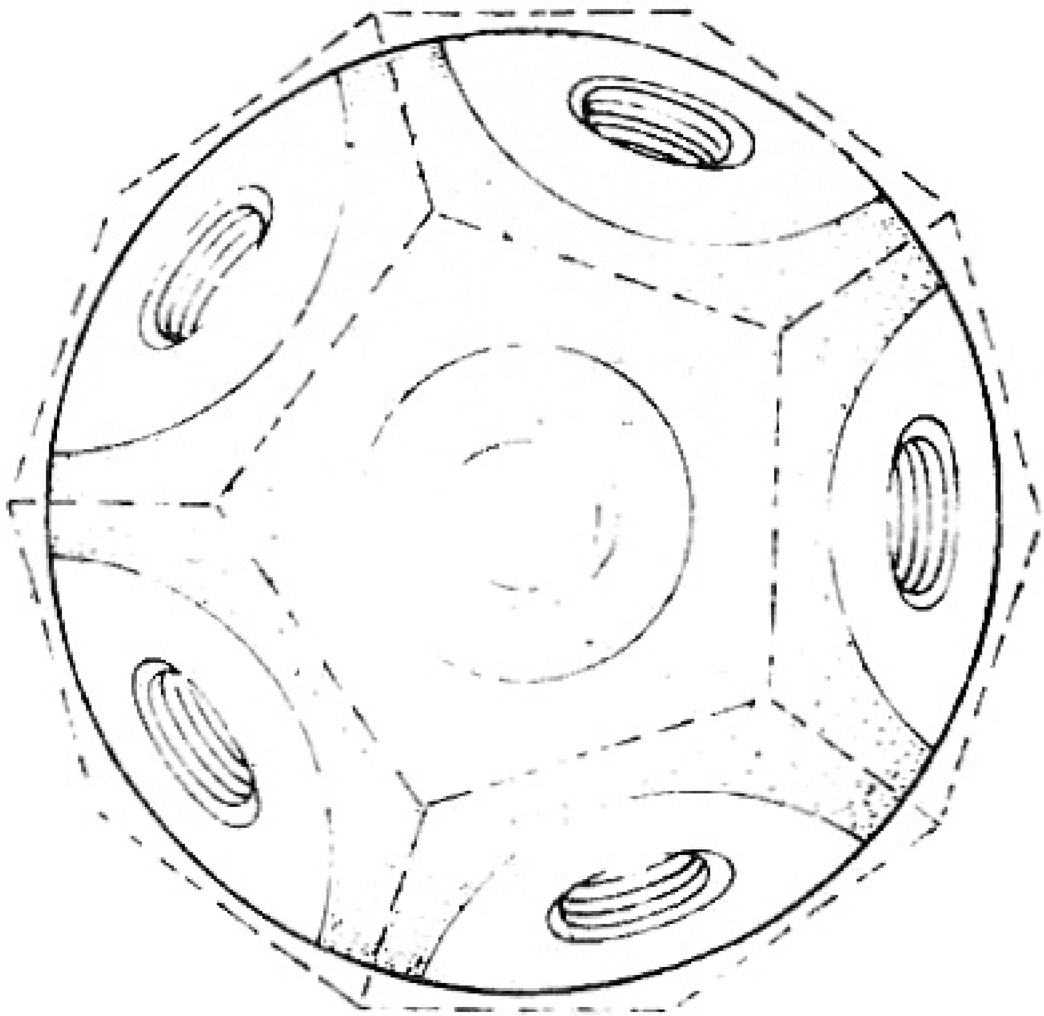


Figure 159: Six Zone Joint Plan

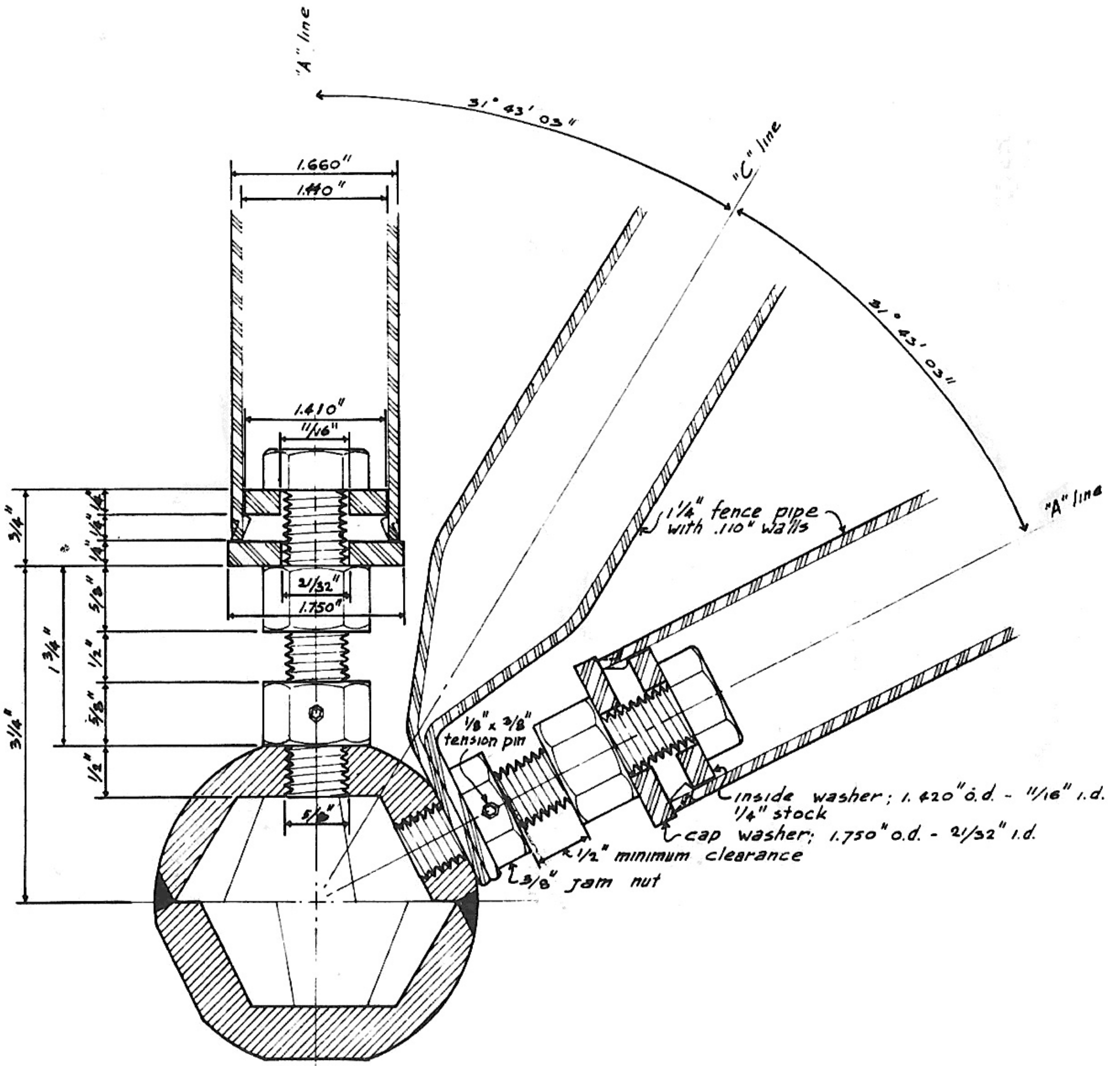


Figure 161: Section for 1¼” pipe

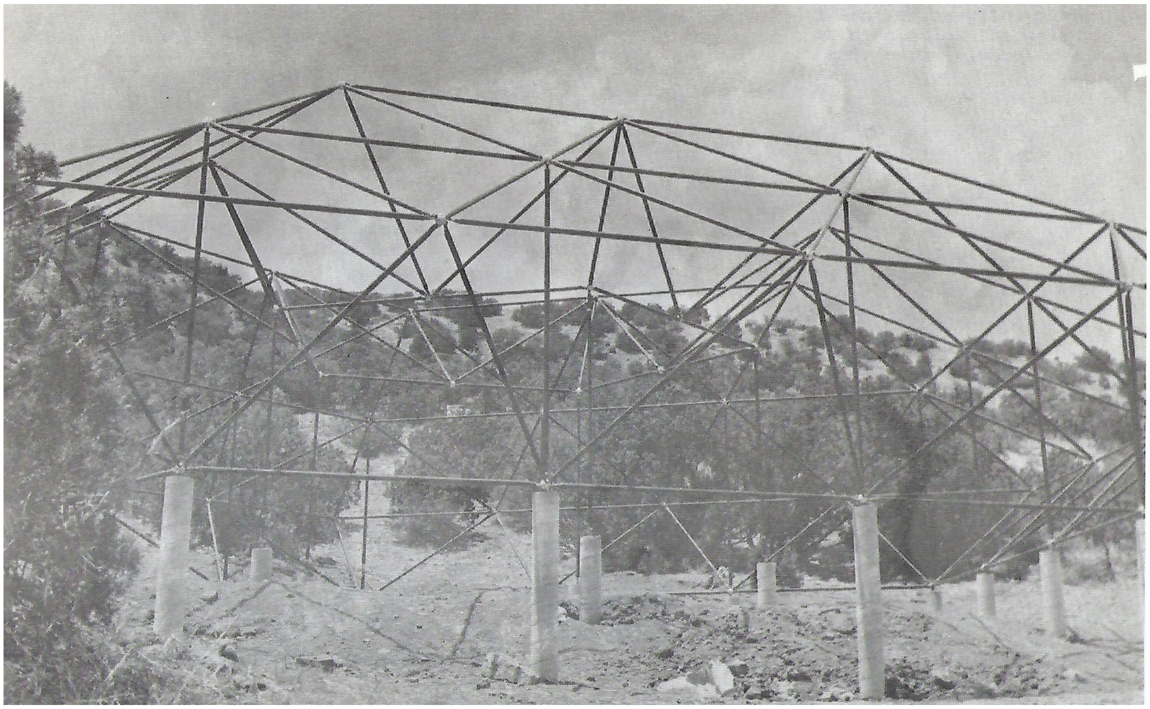


Figure 162: Framework for Robert Ford residence.

A lines = AT^2; base $A = 40''$

C lines $= CT^2$ (see page 32)

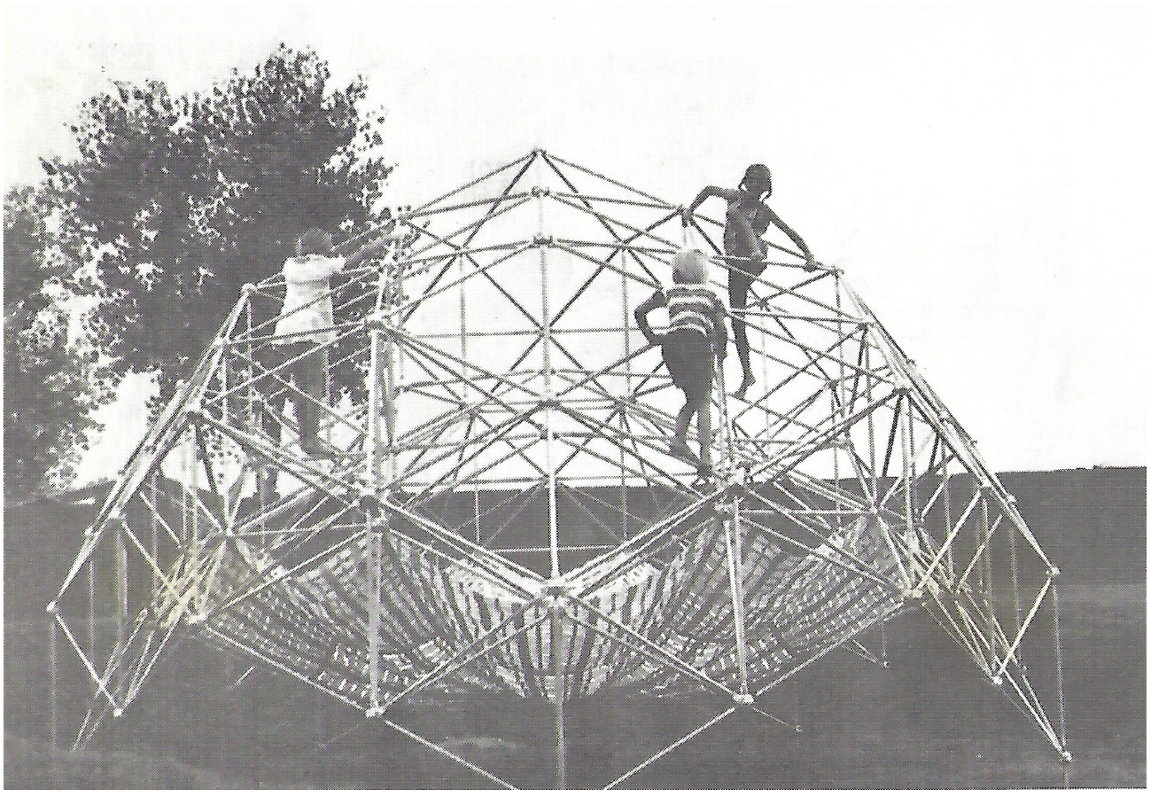


Figure 163: Zome Climber

A lines $= 40''$ C lines $= 42''$

List of Figures

List of Tables

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Index

Todo list